Revealed preferences for diamond goods*

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Abstract

When consumers care for not only the quantity but also the value of a product, it could be rational to purchase products as they become more expensive. This study provides nonparametric—revealed preference—conditions to measure consumers’ marginal willingness to pay for value (i.e., diamondness) associated with particular goods. This is the first nonparametric test of price-dependent preferences. The proposed diamondness measure is applied to observational data from the Russian Longitudinal Monitoring Survey. The results show that this diamondness measure is related to a product’s visibility to society, which indicates a certain degree of conspicuous consumption.

JEL Classification: C14, D03, D11, D12

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1 Introduction

Consumer preferences are characterized by diamond effects—in the spirit of Ng (1987) and Mandel (2009)—when individuals not only care for the quantity of their purchase, but also for the value associated with particular commodities. More specifically, diamond goods (e.g., jewelry) are valued only because they are costly whereas standard goods (e.g., food consumed at home) are valued only for their consumption. This study computes the degree of diamondness, defined as a limit on the willingness to pay for value, of various commodities. Interestingly, diamondness can take any value between 0 (indicating standard goods) and 1 (indicating “pure” diamond goods). Identification of the degree of diamondness is important for several reasons.

First, Ng (1987, 1993) formally analyzes price elasticities and optimal taxation in the presence of one diamond good. The author shows that the rules for optimal indirect taxation depend crucially on a consumer’s marginal willingness to pay for the value of a commodity. From a theoretical perspective, taxing diamond goods increases government revenues without imposing an overly large burden on consumers.

Second, Heffetz (2011) studies heterogeneity in income elasticities across commodities. The budget share spent on food at home (a necessity) typically decreases with income whereas the share spent on food away from home (a luxury good) increases with income. Heffetz (2011) argues that heterogeneity in the shape of Engel functions is, to some extent, predictable. The author uses the visibility of commodities to society to predict income elasticities, which reflects some degree of signaling. Similarly, the diamondness of commodities may shed light on variation in income elasticities and predict elasticities for new products.

Finally, diamond effects that stem from conspicuous consumption motives (Veblen (1899); Bagwell and Bernheim (1996)) and positional concerns (Solnick and Hemenway (1998, 2005); Carlsson et al. (2007)) may lead to overall inefficiency when all individuals spend too much money on the positional commodities while their relative position remains unchanged. Ireland (1994, 1998) and Frank (2008)
discuss policy remedies to address overconsumption of these goods.

This raises the question of which commodities are characterized by high levels of diamondness. Identifying consumers’ marginal willingness to pay for value is not straightforward. The reason is that it is difficult to disentangle price effects through the budget constraint from price effects through preferences.

**Measurement of nonbudget-constraint price effects.** When preferences depend on prices, price variation impacts on consumption outcomes simultaneously via preferences and via the budget constraint. The literature has considered three distinct ways to measure nonbudget-constraint price effects: an experimental approach or a structural (parametric or nonparametric) approach. The first approach is experimental. Heffetz and Shayo (2009) study price-dependent preferences by setting up an experiment in which distinct prices are presented to respondents: relative prices that monitor people’s choice sets and visual price stickers that capture nonbudget-constraint price effects. However, this type of information is unavailable in observational datasets, which are typically used for demand and welfare estimation.

The second approach imposes parametric structure on people’s preferences. Basmann et al. (1988) elicit price-dependent preferences from observational data. To distinguish between price effects through the budget constraint and price effects through preferences, the authors impose a generalized Fechner–Thurstone form on direct utility functions (see also Section 2). This approach, although it is general in the sense that the parameters of the utility function depend on the commodities’ prices, puts much structure on the underlying utility functions. As a result, rejections of rationality may also stem from an incorrect specification of the utility function.

The final option is to use nonparametric—revealed preference—conditions. Revealed preference models in the tradition of Samuelson (1938), Afriat (1967), Dew- ert (1973), and Varian (1982) define refutable conditions that need to hold in order for a consumer to be rational. The conditions are derived from a finite set of observables: selected consumption bundles in different price–income situations. The testable implications are independent of the parametric specification of utility. A crucial assumption in revealed preference applications to longitudinal data is pref-
erence stability. Unfortunately, the specific nature of price-dependent preferences implies that preferences change in response to price variation. For this reason, Bilancini (2011) and Frank and Nagler (2012) correctly argue that revealed preference methods and price-dependent preferences are difficult to reconcile. In particular, Bilancini (2011) points out that revealed preference theory cannot reject behavioral models on the basis of a finite number of observations when preferences depend on budget sets in a general way. Frank and Nagler (2012) add to this that price-dependent preferences are nonetheless crucial predictors of consumer behavior.

Preferences for value. This study shows that revealed preference theory is still useful to study value-dependent preferences, which are a special case of price-dependent preferences. Value-dependent preferences are directly linked to diamond effects—in the narrow sense (Ng (1987, 1993))—although this preference type may also stem from quality considerations (Scitovsky (1945)) and conspicuous consumption (Veblen (1899); Bagwell and Bernheim (1996)). With preferences for value, prices enter the utility function directly as a factor of consumed quantities. This allows for welfare comparisons across periods (given that the utility function itself is “unconditional” on prices) and it still results in testable restrictions for data from budget surveys (given that consumers control the value per commodity by deciding on its consumption). As a result, the marginal willingness to pay for value can be nonparametrically bounded.

More specifically, the revealed preference approach defines personalized prices associated with the intrinsic consumption of commodities and personalized prices associated with the value of commodities. The proposed diamondness parameter bounds the relative magnitude of the latter personalized price, that is, the marginal willingness to pay for value. This methodology associates different levels of diamondness to different commodities, and effectively allows us to investigate the nature of various commodities.

Contributions. This study makes theoretical, methodological, and empirical contributions. First, on the theoretical level, this study introduces a model in which
more than one good can have strictly positive diamondness, and diamondness is expressed in monetary terms of marginal willingness to pay for value. This extends the theoretical model by Ng (1993), which incorporates exactly one diamond good and contains no explicit measure for diamondness. The diamondness parameters in the current study can take any value between 0 (indicating standard goods) and 1 (allowing for pure diamond goods). This approach complies with Rabin (2013)'s portable extensions of existing models (PEEMs) research program, which aims to formulate tractable refinements of existing economic models.

Next, on the methodological level, this study presents revealed preference conditions for consistency with the newly developed behavioral models. This is the first nonparametric test of the utility maximization model with value-dependent preferences. Interestingly, it is shown that the different specifications of preferences for value are generally independent.

Finally, on the empirical level, the approach is demonstrated on the basis of data from the Russian Longitudinal Monitoring Survey (RLMS). In theory, the proposed framework deals with preferences for value driven by quality effects, pure diamond effects, or status (conspicuous consumption) effects. However, the empirical application focuses on the conspicuous consumption source of preferences for value. Indeed, the dataset under consideration contains information on aggregated commodity groups that are heterogeneous in terms of their visibility in society (Heffetz (2011))\(^1\). This provides an intuitive interpretation of the results and sheds light on the underlying drivers of preferences for value in the sample.

The remainder of this paper unfolds as follows. Section 2 defines rationality, preferences for value, and diamondness. Section 3 presents the corresponding revealed preference tests. Section 4 contains a description of the sample taken from the RLMS. Next, Section 5 reports results on the heterogeneity in diamondness across

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\(^1\)Related to this, the aggregates are heterogeneous in terms of their positionality. The utility from positional commodities depends on not only absolute consumption but also relative consumption. The idea is that consumers prefer to have not only a high level of consumption, but also more than others. Solnick and Hemenway (1998, 2005) and Carlsson et al. (2007) shed more light on the positionality of goods. A general finding in this literature is that restaurant visits, clothing, cars, and housing are highly positional commodities.
individuals and, in particular, across commodities. Section 6 concludes.

2 Theory

For each individual consumer, consider a dataset \( S = \{P_t; Q_t\}_{t \in T} \) where \( T \) indicates a set of observations, and let \( N \) indicate a set of commodities. In the empirical application, \(|T| = 11\) and \(|N| = 14\). Then, \( S \) contains information on a series \( t = 1, \ldots, |T| \) of pairwise observations of a quantity vector \( Q_t \in \mathbb{R}^{|N|}_+ \) and a price vector \( P_t \in \mathbb{R}^{|N|}_{++} \). In particular, \( Q_t \) contains the quantities chosen of each commodity in observation \( t \) and \( P_t \) contains the respective prices that define the budget set in observation \( t \). Moreover, let \( M_t \in \mathbb{R}^{|N|}_+ \) represent the vector of expenditure on \(|N|\) commodities in observation \( t \) (i.e., \( M_t \) consists of elements \( M^n_t = P^n_t Q^n_t \)).

**Price-dependent preferences.** Price-dependent preferences, in the spirit of Pollak (1977), imply that consumers care both for the quantity and for the price of their purchase. Higher prices may reflect better quality, a higher financial status and/or simply a higher valuation because of diamondness. The unconditional approach to price-dependent preferences, on the one hand, defines utility as follows:

\[
U(P, Q)
\]

This approach assumes that utility is a function of quantity and prices (Kalman (1968)). This allows for welfare comparisons across periods but it is less well suited to study standard data from budget surveys, in the absence of prices for nonchosen alternatives. The conditional approach to price-dependent preferences, on the other hand, defines utility as follows:

\[
U(Q; P_t)
\]

Consumers care for the quantity of their purchase, given by \( Q \), but this valuation is conditional on the (fixed) set of prices \( P_t \). The main difficulty with this method is its inability to make welfare comparisons across situations with different prices.
(i.e. $P_t$ versus $P_v$) if no parametric structure is imposed. The study by Basmann et al. (1988) circumvents this issue by imposing a specific parametric structure on $U$, which results in a generalized Fechner-Thurstone utility function. The current study proposes an alternative solution. By focusing on preferences for value–rather than price-dependent preferences–testable restrictions are possible even without imposing parametric structure on $U$, thereby setting the stage for revealed preference tests.

**Value-dependent preferences.** A special case of these price-dependent preferences are so called value-dependent preferences, which imply that prices enter the utility function directly as a factor of consumed quantities:

$$U(Q, M)$$

The price vector $P_t$ can affect preferences only via the expenditure vector $M = P_t \odot Q$. First, the form itself of the utility function $U$ does not depend on prices, such that it is possible to make welfare comparisons even when prices change and the functional form of $U$ remains unspecified. Second, the values $M$ are obviously linked to the consumers’ decisions on $Q$, which results in testable restrictions even if the data come from budget surveys (i.e. with a fixed set of prices per choice problem).

The class of value-dependent preferences presented in (1) is a somewhat more general version of the preferences studied by Ng. Ng (1987) first introduced a model with exactly one (pure) diamond good $j$, that is, the market value $M^j$ of this commodity entered the consumer’s utility function: $U(Q^{-j}, M^j)$ with $Q^{-j}$ the quantity vector of all goods except $j$. In a later study, Ng (1993) incorporated one (mixed) diamond good, whereby both the market value $M^j$ and the intrinsic consumption component $Q^j$ of the good entered the utility function: $U(Q, M^j)$. The current model generalizes Ng’s framework, given that it imposes no restrictions on the number of diamond goods.

Other studies in which prices enter the utility function as a factor of quantities include Ng (1987, 1993), Weber (2002), Deng and Ng (2004), Mandel (2009), and Engström (2011).

The symbol $\odot$ denotes the Hadamard product (element-by-element multiplication) of two vectors of the same size.
ber of commodities that are characterized by some degree of diamondness. Second, a new parameter is introduced to capture the degree of diamondness in terms of the marginal willingness to pay for value. This parameter can take any value between 0 (standard good) and 1 (allowing for pure diamond goods), see infra.

By focusing on value-dependent preferences, this study abstracts from price-dependent preferences that take a specific functional form, such as the aforementioned generalized Fechner-Thurstone utility function. This function defines utility as

$$U(Q; P_t) = \prod_{n=1}^{N} (Q^n)^{\beta_n (P^n)^{\sigma}}$$

with preference parameters \(\beta_n\) and \(\sigma\). The prices enter as exponents (rather than factors) of the consumed quantities. Collective models of household consumption are also beyond the scope of this project. Collective models, in the spirit of Chiappori (1988, 1992) and Apps and Rees (1988), assume that households maximize a weighted sum of utility functions \(U^1(Q) + \mu(P_t) U^2(Q)\), with \(U^i(\cdot)\) the utility function of the \(i\)-th member of the household. The (Pareto) weights \(\mu(\cdot)\), which control the intrahousehold distribution of bargaining power, depend on prices \(P_t\). In this case, prices enter the model via a third channel (beside budget constraints and preferences).

**Modeling diamondness.** Utility functions take the form \(U(Q, M)\), implying that consumers derive utility from both quantity and the value of a purchase. In this study, it is required only that \(U(\cdot, \cdot)\) is monotone and concave in its arguments.

$$Q_t = \arg \max_Q U(Q, M) \text{ s.t. } P_t^t Q \leq P_t^t Q_t \text{ and } M = P_t \otimes Q.$$

This yields the following first-order conditions:

$$\forall n \in N : \frac{\partial U(Q_t, M_t)}{\partial Q^n} + \frac{\partial U(Q_t, M_t)}{\partial M^n} P^n_t = \lambda_t P^n_t \quad (2)$$

Expression (2) clearly shows that the total marginal utility from an additional unit of good \(n\) is composed of the marginal utility associated with the good itself \(\left(\frac{\partial U(Q_t, M_t)}{\partial Q^n}\right)\) and the marginal utility associated with the value of this good \(\left(\frac{\partial U(Q_t, M_t)}{\partial M^n}\right)\).
The second term follows directly from the \textit{diamond effect}. Its level is directly related to the magnitude of preferences for value. Let $\theta^n_t$ equal the marginal utility from additional expenditure $\frac{\partial U(Q_t, M_t)}{\partial M^n_t}$ divided by the marginal utility from one unit of income $\lambda_t$. This variable is the main building block of the proposed diamondness measure.

$$\theta^n_t = \frac{1}{\lambda_t} \frac{\partial U(Q_t, M_t)}{\partial M^n_t}$$

Intuitively, $\theta^n_t$ captures the fraction of the marginal utility of income ($\lambda_t$) that stems from preferences for value, rather than preferences for quantity. The variable $\theta^n_t$ is commodity-specific. Certain commodities are more likely to trigger diamond effects than others. Therefore, $\theta^n_t$ can take any value between 0 and 1. First, suppose $\theta^n_t = 1$, that is, the marginal utility from an additional unit of income stems purely from the diamond effect. In this case, good $n$ is a \textit{pure} diamond good (see e.g., Ng (1987)). Consumers derive utility from only the expenditure on this good and not from its quantity. Second, suppose $\theta^n_t = 0$, that is, additional expenditure on good $n$ has no direct impact on the utility of the consumer. In this case, good $n$ is a standard good, in the sense that additional income impacts utility only because the consumer can purchase larger amounts of good $n$. Finally, $\theta^n_t$ can take any value between 0 and 1, that is, $\theta^n_t \in [0, 1]$, which accounts for the possibility that goods are purchased for \textit{both} their consumption and monetary value.

In theory, $\theta^n_t$ can depend on observation $t$. However, it is often more convenient to consider a uniform diamondness per commodity, $\theta^n$, such that $\theta^n_t \leq \theta^n$. The marginal willingness to pay for value $\theta^n_t$ is allowed to vary below $\theta^n$. This simplifies the grid of diamondness values to be considered and improves the empirical bite of the model. For the remainder of this paper, let $\theta$ represent the (column) vector of \textit{uniform} diamondness parameters $\theta^n$ and let $\theta_t$ represent the (column) vector of marginal willingness to pay for value $\theta^n_t$ in observation $t$. Utility maximizing behavior, conditional on the diamondness vector, is described in Problem 1.
Problem 1  Optimization problem \( OPT - \theta \)

\[
Q_t = \arg \max_Q U(Q, M) \text{ s.t. } P_t'Q \leq P_t'Q_t, \ M = P_t \odot Q
\]

\[
\forall n \in N : \frac{1}{\lambda_t} \frac{\partial U(Q_t, M_t)}{\partial M^n} \leq \theta^n
\]  \( (3) \)

Definition 1 links rationalizability with \( \theta - \text{diamondness} \) to consistency with the optimization problem \( OPT - \theta \):

Definition 1  Consider a dataset \( S = \{P_t; Q_t\}_{t \in T} \). Dataset \( S \) is rationalizable with \( \theta - \text{diamondness} \) if there is a monotone and concave utility function \( U(Q, M) \) such that \( \{Q_t\}_{t \in T} \) solves optimization problem \( OPT - \theta \).

The elements \( \theta^n \) of vector \( \theta \) do not necessarily indicate the highest levels of the marginal willingness to pay for value \( \theta^n_t = \frac{1}{\lambda_t} \frac{\partial U(Q_t, M_t)}{\partial M^n} \) that are consistent with Problem 1. Rather, they restrict the range of \( \theta^n_t \) that can be used to rationalize the data. Suppose that \( \theta^n = 0 \) for all \( n \in N \). Then the analysis can only consider standard goods. Suppose on the other hand that \( \theta^n = 1 \). In this case, marginal willingness to pay for value between zero and one may be used. Notice that the diamondness parameter can either be fixed by the researcher (Subsection 5.2) or be endogenous (Subsection 5.1). The former approach is well suited to test the rationality hypothesis conditional on diamondness, i.e. while allowing for a specific degree of preferences for value. The latter approach can be used to recover the minimum level of \( \theta^n \) for which there exist marginal willingness to pay for value that rationalize the data in the sense of Definition 1. At this point, it is worth noting that increasing levels of \( \theta^n \) necessarily lead to more permissive models (as shown by inequality (3)). In this sense, different specifications of \( \theta \) (consisting of elements \( \theta^n \)) are nested. However, the underlying levels of \( \theta^n_t = \frac{1}{\lambda_t} \frac{\partial U(Q_t, M_t)}{\partial M^n} \) are generally nonnested. Attributing a higher level of marginal willingness to pay for value to one particular commodity does not necessarily increase the fit of the model. This independence is discussed in more detail in Subsection 3.2.
3 Methodology

The previous Section 2 formulates a generalization of the original models by Ng (1987, 1993) and proposes a straightforward parameter to capture diamond effects. In the current section, a first nonparametric—revealed preference—test of diamond effects is introduced. The revealed preference approach is particularly attractive in this setting. It avoids putting parametric structure on the utility functions $U(\cdot)$. This guarantees that the recovery of the diamondness vector is independent of the form of utility functions. Moreover, it rules out issues related to unobserved heterogeneity across consumers, since each consumer is analyzed separately.

Contrary to the uniform diamondness parameters $\theta^n$, the underlying marginal willingness' to pay for value $\theta^n_t$ appear to be independent, in general. The section ends with a discussion of empirical performance measures that are standard in the revealed preference literature.

Revealed preference methodology. To set the stage, first consider a revealed preference test for consistency with a monotone and concave utility function of the standard form $U(Q)$. This standard revealed preference test is developed by Afriat (1967), Diewert (1973), and Varian (1982). Specifically, the dataset $S = \{P_t; Q_t\}_{t \in T}$ is said to be rationalizable if there is a monotone and concave utility function $U(Q)$ such that

$$Q_t = \arg\max_Q U(Q) \text{ s.t. } P'_t Q \leq P'_t Q_t$$

No further functional form restrictions are imposed on $U(\cdot)$. The generalized axiom of revealed preference (GARP) provides a straightforward test for the existence of a monotone and concave utility function $U(\cdot)$ that rationalizes the data. The GARP is defined as follows.

Definition 2 (Generalized axiom of revealed preference) Binary relations $R^0_{t,v}$ and $R_{t,v}$ exist such that for all observations $t, v, w$ :

$$R^0_{t,v} \text{ and } R_{t,v}$$
1. \( P'_tQ_t \geq P'_tQ_v \Rightarrow R^0_{t,v} = 1 \),

2. \( R^0_{t,w} = 1 \) and \( R^0_{w,v} = 1 \) \( \Rightarrow \) \( R_{t,v} = 1 \),

3. \( R_{t,v} = 1 \) \( \Rightarrow \) \( P'_vQ_t \geq P'_vQ_v \).

First, the GARP constructs revealed preference relationships \( R^0_{t,v} = 1 \) (observation \( t \) is revealed preferred over observation \( v \)) if \( P'_tQ_t \geq P'_tQ_v \). If \( Q_v \) were affordable at observation \( t \) but not chosen, it must be the case that the consumer preferred \( Q_t \) over \( Q_v \). Second, transitivity is imposed on preferences, such that \( R_{t,v} = 1 \) if \( R^0_{t,w} = 1 \) and \( R^0_{w,v} = 1 \). Finally, the GARP requires \( P'_vQ_t \geq P'_vQ_v \) whenever \( R_{t,v} = 1 \). After all, if bundle \( Q_t \) is revealed preferred over bundle \( Q_v \), it should be chosen over \( Q_v \) whenever it was affordable. In other words, if \( Q_v \) were chosen, it could be inferred that \( Q_t \) was not affordable. Based on this definition, Afriat’s theorem states the following equivalence results.

**Proposition 1** Consider a dataset \( S = \{P_t; Q_t\}_{t \in T} \). The following conditions are equivalent.

1. There is a locally nonsatiated, monotone and concave utility function \( U(\mathbf{Q}) \) such that \( \{Q_t\}_{t \in T} \) solves

\[
Q_t = \arg \max_{\mathbf{Q}} U(\mathbf{Q}) \text{ s.t. } P'_t\mathbf{Q} \leq P'_tQ_t.
\]

2. For all decision situations \( t \in T \), there are utility numbers \( u_t \) and (Lagrange) multipliers \( \lambda_t > 0 \) such that for all \( t, v \in T \):

\[
u_t - u_v \leq \lambda_v P'_v(Q_t - Q_v).
\]

3. For all decision situations \( t \in T \) : \( S = \{P_t; Q_t\}_{t \in T} \) satisfies the GARP.

Statement 2 contains the so-called Afriat inequalities. Observed behavior can be rationalized by the standard utility maximization model if and only if the data are consistent with the Afriat inequalities. Moreover, these conditions are equivalent to stating that \( S = \{P_t; Q_t\}_{t \in T} \) is consistent with the GARP.
3.1 Revealing preferences for diamond goods

The aforementioned revealed preference conditions can be used to verify the existence of a utility function $U(Q)$ of the standard form. To reveal preferences for diamond goods, a modification is necessary. In particular, the question is whether there is a well-behaved utility function $U(Q, M)$ such that the observed consumption pattern $\{Q_t\} t \in T$ solves Problem $OPT − \theta$. The desired consistency test is independent of parametric structure on the utility functions $U(·, ·)$. It is assumed only that utility functions are monotone (nondecreasing) and concave in their respective arguments.

Furthermore, the specific focus on diamond effects (i.e. value-dependent preferences) makes that it remains possible to have a homogeneous preference ordering over quantity and value\(^4\) while still leading to testable implications when applied to data from standard budget surveys. Indeed, consistency of the observed choices can still be verified using first-order conditions, and marginal utilities from value can be nonparametrically bounded. As a result, the revealed preference approach is both valid and useful for testing consistency with the utility maximization hypothesis in the presence of diamond effects.

Starting from concavity of the utility function, the following inequalities must hold for all $t, v \in T$:

$$U(Q_t, M_t) − U(Q_v, M_v) \leq \sum_{n=1}^{N} \frac{\partial U(Q_v, M_v)}{\partial Q_n} (Q_t^n - Q_v^n) + \sum_{n=1}^{N} \frac{\partial U(Q_v, M_v)}{\partial M_n} (P_t^n Q_t^n - P_v^n Q_v^n)$$

The first-order conditions associated with Definition 1 have been presented in equation (2). They imply that

$$\lambda_t P_t^n = \frac{\partial U(Q_t, M_t)}{\partial Q_n} + \frac{\partial U(Q_t, M_t)}{\partial M_n} P_t^n$$

\(^4\)Provided that the social environment is rather stable.
Given the definition of $\theta^t_n = \frac{1}{\lambda_t} \frac{\partial U(Q_t, M_t)}{\partial M^n}$, it follows that

$$\frac{\partial U(Q_t, M_t)}{\partial Q^n} = \lambda_t (1 - \theta^n_t) P^n_t$$

Moreover, $u_t = U(Q_t, M_t)$, $u_v = U(Q_v, M_v)$ and column vector $\theta_t$ contains the marginal willingness' to pay for value associated with $|N|$ commodities at observation $t$. The combination of the above restrictions yields necessary conditions for rationalizability with $\theta - diamondness$. Appendix A shows that these conditions are also sufficient. Proposition 2 then presents two straightforward tests of consistency with $\theta - diamondness$.

**Proposition 2** Consider a dataset $S = \{P_t; Q_t\}_{t \in T}$ and $\theta \in [0, 1]$. The following conditions are equivalent:

1. The dataset $S$ is rationalizable with $\theta - diamondness$.

2. For all decision situations $t \in T$, there are utility numbers $u_t$, (Lagrange) multipliers $\lambda_t > 0$ and shadow prices $0 \leq \theta^n_t \leq \theta^n$ and $\pi_t^n = (1 - \theta^n_t) P^n_t$ such that for all $t, v \in T$:

$$u_t - u_v \leq \lambda_v \sum_{n=1}^{\mid N \mid} \pi_v^n (Q^n_t - Q^n_v) + \lambda_v \sum_{n=1}^{\mid N \mid} \theta^n_v (P^n_t Q^n_t - P^n_v Q^n_v).$$

3. For all decision situations $t \in T$, there are shadow price vectors $0 \leq \theta_t \leq \theta$ and $\pi_t = (1 - \theta_t) \otimes P_t$ such that $\tilde{S} = \{\pi_t, \theta_t; Q_t, M_t\}_{t \in T}$ satisfies the GARP. That is, there are binary relations $R^0_{t,v}$ and $R_{t,v}$ such that for all $t, v, w \in T$:

   (a) $\pi'_t Q_t + \theta'_t M_t \geq \pi'_v Q_v + \theta'_v M_v \Rightarrow R^0_{t,v} = 1,$

   (b) $R^0_{t,w} = 1$ and $R^0_{w,v} = 1 \Rightarrow R_{t,v} = 1,$

   (c) $R_{t,v} = 1 \Rightarrow \pi'_v Q_t + \theta'_v M_t \geq \pi'_v Q_v + \theta'_v M_v.$
Statement 1 defines rationality when the magnitude of the diamond effects is bounded by $\theta$. Statement 2 presents a test of $\theta$ – *diamondness* based on inequalities that are similar to the well-known Afriat inequalities. However, there are two main differences between these inequalities and the Afriat inequalities. First, additional arguments—related to the expenditure $M_v$—enter the inequalities. Second, the market prices $P_v$ are replaced with shadow prices $\pi_v = (1 - \theta_v) \odot P_v$—associated with quantity—and $\theta_v$—associated with value. Statement 3 presents an equivalent test of $\theta$ – *diamondness* based on the GARP conditions.

**Implementation and endogeneity of diamondness.** Statements 2 and 3 in Proposition 2 are easily implementable. Conditional on shadow price vectors $\theta_t$, the conditions in Statement 2 are linear in $u_t$, $u_v$, and $\lambda_v$. Therefore, linear programming techniques are well suited to implement this test. Statement 3 contains the alternative GARP formulation. This formulation is particularly convenient. First, testing consistency with Statement 3 simply requires the verification of a set of combinatorial restrictions when the shadow price vectors $\theta_t$ are given. It is no longer necessary to formulate and solve a programming problem.

Of course, the set of marginal willingness to pay for value is generally unknown and not observable. The GARP-based test, presented in Statement 3, can be implemented using a linear programming problem with binary variables even if the shadow prices $\theta^n_t$ (and $\pi^n_v$) are unobserved and unspecified. The key innovation of GARP is that the variables $u_t$, $u_v$, and $\lambda_v$ are replaced with binary variables $x_{t,v}$ indicating $x_{t,v} = 1$ when $u_t \geq u_v$ and $x_{t,v} = 0$ otherwise. The mixed integer linear programming implementation of GARP-based conditions is clearly illustrated in Cherchye et al. (2011).

Finally, recall that $\theta^n$ serves as an upper bound on shadow prices $\theta^n_t$. Given that $\theta^n$ can be endogenous, the corresponding $\theta^n$ can also be computed. In particular, one can identify the minimum level of diamondness $\theta^n$ such that there exist values $\theta^n_t$ that rationalize the data. Higher levels of $\theta^n_t = \frac{1}{\lambda_t} \frac{\partial U(Q, M_t)}{\partial M_t}$ do not necessarily improve the fit of the model (Subsection 3.2) but the goodness-of-fit is always (weakly) increasing in the upper bound $\theta^n$. 

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Negative marginal utility. Incidentally, the presented framework is also useful for the analysis of “bads”. The standard neoclassical model stipulates that consumers should not spend their budgets on bad commodities, which reduce consumers’ intrinsic utility. However, when preferences depend on value, a rational consumer could purchase additional units of a bad, as long as the marginal utility from its value exceeds the negative intrinsic marginal utility. Testing whether a commodity \( n \) is a bad is now easy. The requirement \( \theta_{v}^{n} \leq \theta^{n} \leq 1 \) can simply be replaced with \[ \theta_{v}^{n} > 1 \]

in the revealed preference characterization in Proposition 2. Then, it is easy to verify that \[ \pi_{v}^{n} = \frac{1}{\lambda_{v}} \frac{\partial U(Q_{v}, M_{v})}{\partial Q^{n}_{v}} = (1 - \theta_{v}^{n}) P_{v}^{n} < 0. \] The marginal utility from quantity can be negative as long as this negative effect is offset by the corresponding positive marginal utility from value.

3.2 Graphical illustrations of independence

It was already mentioned that the marginal willingness’ to pay for value \( \theta_{t}^{n} = \frac{1}{\lambda_{t}} \frac{\partial U(Q_{t}, M_{t})}{\partial M^{n}_{t}} \) are generally nonnested. This implies that higher degrees of \( \theta_{t}^{n} \) do not necessarily allow for more degrees of freedom in consumption (in contrast to higher levels of \( \theta^{n} \)). Indeed, it is possible that a dataset is consistent with the standard model whereas it is not consistent with some strictly positive specification of \( \theta_{t}^{n} \). Likewise, a dataset may simultaneously violate the standard model and satisfy the model with strictly positive \( \theta_{t}^{n} \). This suggests that the set of consistent marginal willingness’ to pay for value may be bounded both from below and from above.

To demonstrate the independence of different specifications, consider the graphical illustrations of two (distinct) datasets in Figures 1 and 2. Consider first a dataset with \( |T| = 2 \) observations, \( |N| = 2 \) goods, and price vectors \( P_{1} = (2; 1) \) and \( P_{2} = (1/2; 1) \) and quantity vectors \( Q_{1} = (3; 2) \) and \( Q_{2} = (2; 3) \) for the first and second observations, respectively. The left graph in Figure 1 presents the corresponding budget constraints in terms of quantities. After all, consumers who have standard preferences care only for consumed quantities. Given this assumption, however, the
choices in this first example are clearly irrational. The consumer preferred bundle $Q_1$ over bundle $Q_2$ in the first observation while he preferred $Q_2$ over $Q_1$ in the second observation. It is impossible to construct noncrossing indifference curves from a standard well-behaved utility function through the chosen bundles. The right graph in Figure 1 differs from the left in that good 1 is presented as a (pure) diamond good, that is, with $\theta_1^1 = \theta_2^1 = 1$. As such, consumers care for the quantity associated with good 2 and the value associated with good 1. The budget constraints are redefined accordingly. The budget constraint of observation 1 tilts outward because the quantities of good 1 are multiplied by the high price of good 1 in observation 1. Similarly, the budget constraint of observation 2 tilts inward because the quantities of good 1 are multiplied by the low price of good 1 in observation 2. Given this alternative assumption driven by the diamondness of good 1, the consumer still preferred bundle $Q_1 = (M_1^1; Q_2^1)$ over bundle $Q_2 = (M_2^1; Q_2^2)$, but the reverse no longer holds. There are indifference curves through the chosen bundles that do not cross. The choices are perfectly rationalizable when good 1 is a diamond good$^5$. The observed expenditure pattern is consistent with any preference ordering $R$—defined over the money spent on diamond good 1 and the quantity associated with standard good 2—that ranks bundle $Q_1$ higher than bundle $Q_2$. Intuitively, the consumer spent too much money on good 1 in observation 1 conditional on standard preferences, but this expenditure pattern is explained by the fact that the consumer desired the (high) value of good 1.

For the reverse, consider a dataset with $|T| = 2$ observations, $|N| = 2$ goods, and price vectors $P_1 = (2; 1)$ and $P_2 = (1; 2)$ and quantity vectors $Q_1 = (3; 0)$ and $Q_2 = (2; 3)$ for the first and second observations, respectively. The left graph in Figure 2 presents the budget constraints in terms of quantities. The chosen bundles clearly correspond to rational behavior, given the standard assumption of preferences for quantity. The consumer preferred bundle $Q_2$ over bundle $Q_1$ in the second observation, while $Q_2$ was not affordable in the first observation. The observed expenditure

$^5$Algebraically, it can be verified that $|P_1'Q_1 = 8| > |P_1'Q_2 = 7| \Rightarrow u_1 > u_2$ and $|P_2'Q_2 = 4| > |P_2'Q_1 = 3.5| \Rightarrow u_2 > u_1$ lead to a contradiction, while the combination of $|P_1'Q_1 = 8| > |M_1^1 + P_2'Q_2 = 4| \Rightarrow u_1 > u_2$ and $|P_2'Q_2 = 4| < |M_2^1 + P_2'Q_1 = 8| \Rightarrow u_2 > u_1$ is feasible.
pattern is consistent with any preference ordering $R'\sim$—defined over the consumed quantities of commodities 1 and 2—that ranks bundle $Q_2$ higher than bundle $Q_1$. The right graph in Figure 2 differs from the left in that good 1 is presented as a pure diamond good, that is, with $\theta_1^1 = \theta_2^1 = 1$. As such, consumers care for the quantity associated with good 2 and the value associated with good 1. The budget constraints are redefined accordingly. Specifically, the budget line associated with observation 1 tilts outward because the prices of good 1 are taken up as arguments of the utility function. However, given this alternative assumption driven by the diamondness of good 1, the choices are no longer consistent. Strong preferences for the value of good 1 in observation 1 imply that $\tilde{Q}_1 = (M_1^1; Q_1^2)$ is a better bundle than $\tilde{Q}_2 = (M_2^1; Q_2^2)$, which contradicts the fact that $\tilde{Q}_1$ was available in observation 2 but yet not chosen. It is no longer possible to construct well-behaved indifference curves through the chosen bundles in such a way that the curves do not cross. As a result the theory of rational consumption rejects good 1 being a pure diamond good (i.e., $\theta_1^1 = \theta_2^1 = 1$ is infeasible)\(^6\).

Finally, it is worth noting that allowing for preferences for value also reduces the

\(^6\)Algebraically, it can be verified that the combination of $[P^1_1 Q_1 = 6] < [P^1_2 Q_2 = 7] \Rightarrow u_1 > u_2$ and $[P^2_2 Q_2 = 8] > [P^2_2 Q_1 = 3] \Rightarrow u_2 > u_1$ is feasible, while $[P^1_1 Q_1 = 6] > [M_2^1 + P^2_1 Q_2^2 = 5] \Rightarrow u_1 > u_2$ and $[P^2_2 Q_2 = 8] > [M_1^1 + P^2_2 Q_1^1 = 6] \Rightarrow u_2 > u_1$ lead to a contradiction.
level of price variation in the data. Given full diamondness $\theta^n_t = 1$, the shadow prices at which expenditure on commodity $n$ is valued equal 1, because variation in the market price of the corresponding commodity is included as a factor of quantity. This is reflected in Figures 1 and 2. In these figures, the graphs on the right, which take into account that good 1 is a diamond good, show considerably less variation in the slopes of budget lines compared to the graphs on the left. It is well known that revealed preference methods are more “powerful” when budget lines cross frequently, that is, if there is sufficient relative price variation. As a further implication, rationality can never be rejected by a specification that treats all goods as “pure” diamond goods\(^7\) (i.e., $\theta^n_t = 1$ for all $n \in N$ and $t \in T$). An empirical solution to these problems is discussed in the following Subsection 3.3. Specifically, discriminatory power captures the strength of a test, that is, its ability to reject rationality of random, simulated datasets.

\(^7\)This result is derived from the Afriat-like inequalities in Proposition 2. First, $\theta^n_t = 1$ ($\forall n \in N$ and $\forall t \in T$) implies that

$$u_t - u_v \leq \lambda_v \sum_{n=1}^{N} (P^n_t Q^n_t - P^n_v Q^n_v)$$

This set of conditions is trivially satisfied with equality if $\forall t, v \in T : u_t = P'_t Q_t, u_v = P'_v Q_v$ and $\lambda_v = 1$. 

![Figure 2: Independence example: inconsistent diamondness (left graph: all standard goods, right graph: diamond good 1)](image-url)
3.3 Measures of empirical performance

The empirical performance of different specifications of $\theta$ can be examined by standard measures in the revealed preference literature. Three commonly used criteria, in this respect, are pass rates, power, and predictive success.

The revealed preference test analyzes each consumer separately. The test gives a positive response (1) if the individual’s behavior can be rationalized by a well-behaved utility function, and a negative response (0) if the individual’s behavior cannot be rationalized—conditional on some specification of $\theta$. The average response (“pass rate” $r$) equals the fraction of observed datasets that can be rationalized. Hence, a pass rate of 1 indicates that each individual has consistently chosen according to the specified model whereas a pass rate of 0 indicates that no agent is rational according to the model.

However, in order to select the “best” specification of $\theta$, the strength of the corresponding test is also important. Increasing the diamondness (i.e. the upper bound on marginal willingness’ to pay for value) naturally increases a model’s fit—leading to better pass rates—but also makes the model more permissive.$^8$ Permissive models typically lack the power to distinguish between actual consumer behavior and random, simulated data. One way of controlling the permissiveness of the testable implications is to study “discriminatory power,” $d$ (for a review of power measures, see Andreoni et al. (2011)). The power of a revealed preference test is the probability that the null hypothesis of optimizing behavior is rejected when some alternative hypothesis is true. The alternative hypothesis may stipulate that budget shares are drawn at random from the empirical distribution of budget shares. This corresponds to the so-called bootstrap power index. Another alternative hypothesis is that budget shares are drawn at random from a uniform distribution on the unit interval. This is the power index of Bronars (1987). Discriminatory power is one minus the pass rates of datasets constructed under these alternative hypotheses. The more powerful

$^8$Although the levels of marginal willingness to pay for value $\theta^n$ are generally independent (i.e. nonnested), it was discussed in Subsection 3.2 that controlling for power is necessary also for these variables. After all, higher marginal willingness’ to pay for value may decrease the probability that budget lines intersect and thereby weaken the testable implications of the model.
specifications of the model are less permissive and therefore, are desirable.

Ideally, a specification of $\theta$ combines a very high pass rate ($r$) with a very high discriminatory power ($d$). Predictive success ($p$)—proposed by Selten (1991) and discussed in more detail by Beatty and Crawford (2011)—summarizes the information on pass rates and power. In other words, predictive success ($p$) is a power-adjusted pass rate. It is defined as:

$$p = r - (1 - d)$$

Higher predictive success indicates that the model is better able to distinguish between observed behavior (which is supposed to be rational according to the model) and random, simulated behavior (which is supposed to violate the conditions of the model). Hence, a predictive success of 1 suggests that each observed dataset is consistent with the model whereas each random dataset violates the conditions of the model. A predictive success of $-1$ is the worst possible outcome, while a predictive success of 0 implies that the model cannot discriminate between real and random datasets. Hence, positive predictive success scores are desirable.

4 Data

The consumer data in this study are taken from the RLMS, from 1994 to 2006, with the exception of 1997 and 1999. These 11 waves correspond to the second collection phase of the RLMS data (Phase II).

Attention is restricted to single people who do not receive any unemployment benefits. Furthermore, the sample consists only of single people who report expenditure for the 11 waves. Finally, all single people were house and car owners during the full period of observation. This leads to a sample of 82 single people.

“Big” decisions on durable goods are excluded from the analysis by condition-

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9Thus, we implicitly assume that decisions on these nondurable commodities and “big” decisions on durables are weakly separable. Although this assumption is contestable, it is quite common in applied static demand analysis. Moreover, interpersonal variation in durable decisions is not an issue, since each agent is analyzed separately.
ing on house and car ownership. The reason is straightforward: it is important to distinguish between decisions driven by diamond effects on the one hand and intertemporal portfolio decisions on the other hand. Since the focus of this study is on the former, durable commodities were dropped.

The nondurable commodities in the sample are bread, potatoes, vegetables, fruit, meat, dairy products, alcohol, tobacco, food outside home, clothes, car fuel, wood fuel, gas fuel, and luxury products. The aggregates under consideration are heterogeneous in terms of their visibility to society. This allows for a study of the relationship between diamondness and the visibility index of Heffetz (2011). Moreover, this grouping follows Cherchye et al. (2009), who conduct a similar revealed preference application to the RLMS data. The aggregation procedure in this study is very similar\(^{10}\).

Specifically, aggregation in the current study avoids unit values by defining aggregate prices as a weighted geometric mean of the prices associated with goods in the corresponding subgroup. For instance, the price of alcohol is a weighted mean of the prices of vodka, liquor, and beer. Therefore, the aggregate price \(P^n_v\) for some commodity group \(n\) in period \(v\) is a weighted Jevons-type price index:

\[
P^n_v = \prod_{k} (p^k_v)^{w^k}
\]

where \(k\) indexes the subgroups of products that belong to the aggregate commodity \(n\), and the weights \(w^k\) are determined by the average expenditure share of \(k\) relative to the expenditure on the commodity \(n\). Because the weights \(w^k\) are fixed, the aggregate prices are independent of variation in the quality composition of the aggregate. Subsequently, the aggregate quantities are derived from the total expenditure and the aggregate prices. At this point, it is noteworthy that the aggregation may still underestimate demand responses to changing prices. When one particular good comprised in an aggregate becomes more expensive and an individual consumes more of this good relative to the average expenditure share, the aggregate

\(^{10}\)Notice that aggregation raises the issue of measurement error. See Varian (1985) for a discussion of measurement error in a revealed preference analysis.
quantity for the individual is overestimated. It may seem that the individual did not react strongly to the change in price. One possibility is to rely on Hicks’ composite commodity theorem, which implies that testing consistency on the basis of commodities $Q^n_v$ and aggregate prices $P^n_v$ is equivalent to testing consistency on the basis of goods $q^l_v$ and $q^h_v$ and associated prices $p^l_v$ and $p^h_v$, respectively, provided that the relative prices $p^h_v/p^l_v$ remain stable across observations $v$ (i.e., $p^h_v/p^l_v = \alpha$). Appendix B shows that Hicks’ theorem still holds for the aggregation method at hand—provided that the diamondness does not vary within the aggregates. If the theorem does not hold, aggregation issues may impact on the rationality results. Because the aggregation issue applies to both visible and less visible goods, this may be expected to weaken the link between the visibility of commodities and our diamondness measure. However, Section 5 shows that the relationship is apparent, and rather strong.

For the revealed preference analysis, attention is restricted to real prices. All nominal prices are divided by the average price level in each period. In this way, price changes due to inflation have no impact on the consumption decisions of consumers. The implicit assumption that there is no money illusion is standard in many studies that involve price-dependent preferences.

In order to limit the number of parameters to be estimated in the empirical application, diamondness varies only across four commodity groups: food at home (bread, potatoes, vegetables, fruit, meat, and dairy products), food away from home (alcohol, tobacco, and other food outside home), clothing/luxuries (clothing and jewelry), and fuel.

Moreover, food at home and fuel can be distinguished from food away from home and clothing/luxuries on the basis of a visibility ranking created by Heffetz (2011). To create this ranking, Heffetz (2011) uses information from 480 interviews on the visibility of various commodities. The main question in Heffetz’ survey is whether respondents would notice if another household spent more than average on some commodity (e.g., jewelry and watches). In addition, respondents were asked how much time it would take to notice this more-than-average spending pattern. In this way, the commodities (not brands) that were most visible to society were expected to obtain a high rank.
Tobacco (ranked 1st), clothing (3rd), jewelry (5th), food outside home (7th), and alcohol (8th) were all part of the top-10 most visible commodities, according to Hef-fetz (2011)’s ranking. Therefore, food away from home and clothing/luxuries were assigned to the “visible” category. Food at home (14th) and fuel (gasoline ranked 21st, and utilities 25th) were ranked considerably lower. These commodities were collected in the “invisible” category. Although this study does not formally disentangle different sources of price-dependent preferences, the information on the visibility of commodities provides an interesting interpretation of the results.

As a final note, powerful tests of revealed preference theory typically require sufficient relative price variation but stable preferences. Figure 3 presents the evolution of prices and budget shares from reference year 1994 to 2006. First, the data contain considerable relative price variation. Prices of food away from home, clothing, luxuries, and fuel increased significantly in relative terms while prices of food at home remained more or less constant. Second, relative expenditure on clothing peaked in 2001 whereas relative expenditure on fuel peaked in 2006. Food expenditure (both at home and away), on the other hand, remained relatively stable. One particular concern is that strong correlations between prices and budget shares of the visible commodities reflect changing preferences over time rather than diamond effects. However, the specific patterns of expenditure on the visible commodities (food away from home and clothing) show no strongly monotone increase over the years.

5 Results

First, this section presents the distribution across all individuals of the average diamondness necessary to rationalize the observed consumption. This gives insight into the required deviation from the standard model to describe each consumer’s spending pattern. The distinction is made between recovery of the diamondness parameter \( \theta^n \) and recovery of the marginal willingness to pay for value \( \theta^n \leq \theta^n \).

Second, the focus shifts from interindividual heterogeneity in preferences for value to heterogeneity in the diamondness across different commodities. In partic-
Figure 3: Evolution of budget shares and prices

5.1 Interpersonal heterogeneity in preferences for value

Distribution of diamondness. The exercises that are presented in this subsection address heterogeneity in preferences for value across individuals, and are mainly exploratory in nature. The aim of this first exercise is to recover the distribution of diamondness in the sample. Similar to heterogeneity in standard preferences, one may also expect considerable heterogeneity in the preferences for value. Given this focus on interpersonal variation in diamondness, one diamondness measure per individual is recovered. This measure is the mean diamondness (per individual) across
all commodities. Formally,

\[ \bar{\theta} = \min_{\theta^n, \bar{\theta}^n, \pi_t^n} \sum_{n=1}^{\left| N \right|} \varpi^n \theta^n \]

s.t.

\[ \bar{\theta} = \{ \pi_t, \theta_t; Q_t, M_t \}_{t \in T} \text{ satisfies the GARP}, \]

\[ \theta_t^n \leq \bar{\theta}^n \text{ and } \pi_t^n = (1 - \theta_t^n) P_t^n . \]

Recall that the diamondness parameter \( \theta^n \) provides a technical upper bound on the marginal willingness to pay for value. Furthermore, \( \sum_{n=1}^{\left| N \right|} \varpi^n \theta^n \) is the mean diamondness across all goods, with weights \( \varpi^n \) given by the corresponding expenditure shares. Finally, \( \bar{\theta} \) captures the minimum level of (average) diamondness that is required to rationalize the data.\(^{11}\) That is, the data cannot be rationalized when the (average) limit on the marginal willingness to pay for value is below \( \bar{\theta} \). To compute this diamondness, it suffices to add an objective function to each individual’s linear programming problem with binary variables, derived from Statement 3 in Proposition 2. Figure 4 shows the distribution of \( \bar{\theta} \).

It is clear from Figure 4 that around 56 per cent of the individuals behave consistently with the classical model. The null hypothesis of standard preferences for quantity is not rejected for these individuals. However, up to 85 per cent of the individuals are rational when \( \bar{\theta} = 0.3 \). It appears that the behavior of many additional consumers can be described by allowing for some—albeit not maximal—diamondness.

The heterogeneity in \( \bar{\theta} \) in Figure 4 reflects preference heterogeneity in the sample. This raises the question whether people from different social classes also have different preferences for value. To address this question, Figure 5 plots the distribution of diamondness separately for individuals whose total expenditure is below the 33—th percentile and individuals whose total expenditure is above the 67—th percentile. There appears to be no significant difference between the distributions.

\(^{11}\) Recall that any dataset is trivially rationalized when the diamondness of all commodities is 1.
Figure 4: Cumulative distribution of the minimum average diamondness \( \bar{\theta} \) necessary to rationalize the data.

Figure 5: Cumulative distribution of the minimum average diamondness \( \bar{\theta} \) necessary to rationalize the data, conditional on total expenditure (low expenditure: below 33–th percentile; high expenditure: above 67–th percentile).
Nevertheless, a fraction of low-income individuals is characterized by the highest levels of diamondness. This suggests that individuals at the bottom of the income distribution have systematically different preferences to others. Banerjee and Duflo (2007) analyze the economic lives of the extremely poor, and find, quite surprisingly, that the poor spend much money on festivals (e.g., a wedding or funeral) at the expense of lower food consumption and health. A possible explanation of these phenomena is provided by Veblen (1899), who argues that members from a lower class acquire expensive commodities in order to boost their social status. On the one hand, Bagwell and Bernheim (1996) interpret Veblen’s (1899) ideas in a model of wealth signaling. On the other hand, Sivanathan and Pettit (2010) provide an alternative psychological explanation of the excessive consumption of status goods by individuals of lower classes. The authors find that especially individuals’ lowered self-esteem drives their willingness to spend money on status goods. In other words, these individuals engage in excessive status consumption for reasons of self-integrity and affirmation.

**Set-identification of willingness to pay for value.** The previous exercise clearly demonstrates that there is variation in the preferences for value across different individuals. However, as indicated earlier, the diamondness parameters impose uniform upper bounds on the marginal willingness’ to pay for value that can be considered to rationalize the data. So, while these upper bounds may be different, the true underlying marginal willingness to pay for value may be similar across individuals. For this reason, the second exploratory exercise recovers all levels of marginal willingness to pay for value \( \theta_t \) that can rationalize the data. Notice that the sets of feasible levels of \( \theta_t \) are not necessarily convex. To keep the exposition simple, this exercise focuses on the marginal willingness to pay for value associated with one particular subset of commodities, i.e. the more visible goods. In the following subsection, it is shown in detail that this category is more likely to produce strong preferences for value.

Formally, define \( \tilde{\theta} = \frac{1}{|T|} \sum_{t=1}^{|T|} \delta_t \) with \( \delta_t \) the marginal willingness to pay for value
associated with visible consumption and \( \tilde{\theta} \) the mean value across all observations of the individual under consideration. Suppose that \( \tilde{\theta} \) belongs to set \( \Theta \) if it can be constructed as the mean value over a set of willingness’ to pay \( \delta_t \) that rationalize the data, i.e.

1. \( \tilde{\theta} = \frac{1}{|T|} \sum_{t=1}^{|T|} \delta_t \)

2. \( S = \{ \pi_t, \theta_t; Q_t, M_t \}_{t \in T} \) satisfies the GARP

3. \( \left[ \begin{array}{c} \theta^n_t = \delta_t \text{ if } n \text{ is visible;} \\ \theta^n_t = 0 \text{ otherwise.} \end{array} \right. \) and \( \pi_t^n = (1 - \theta^n_t) P^n_t \).

In addition, the prices \( \delta_t \) are restricted to vary close to the mean \( (0.9 \delta_t \leq \tilde{\theta} \text{ and } 1.1 \delta_t \geq \tilde{\theta}) \) in order to improve the empirical bite of the test. Finally, \( \Theta \) is the feasible region of the mean marginal willingness’ to pay for value (associated with visible commodities). Figure 6 presents, per individual, the set \( \Theta \) (in black). The individuals are ordered in terms of their total expenditure; individuals at the lower end of the expenditure distribution are on the left in Figure 6.

Many individuals behave in a way that is either rationalizable or not rationalizable, regardless of the specification of preferences for value\(^{12}\). However, there is also some heterogeneity across the sample. In general, the black area (rationalizable area) is somewhat more prominent at higher levels of \( \tilde{\theta} \). Furthermore, the results in Figure 6 show that, contrary to the diamondness bounds \( \theta^n \) (and the average \( \tilde{\theta} \)), different levels of \( \theta^n_t \) (and the average \( \tilde{\theta} \)) are nonnested. For two individuals, \( \Theta \) is bounded from above by 0.62 and 0.84, respectively. The behavior of these consumers is consistent with the standard GARP (i.e. Definition 2) but not with the revealed preference test based on strong preferences for value. For another individual, the region of feasible values is even nonconvex: \( \Theta = [0, 0.18] \cup [0.56, 1] \). This shows that the independence result from Subsection 3.2 is not merely a theoretical curiosum but also an empirical result.

\(^{12}\)In this respect, it is noteworthy that the diamondness parameter is not identified for individuals whose preferences are represented by a Cobb–Douglas utility function. The reason is that Cobb-Douglas expenditure shares are constant (and independent of prices) regardless of the specification of \( \theta_t \) (and \( \theta \)).
singles willingness to pay for value associated with visible goods $\bar{\theta}$

Figure 6: Heterogeneity in $\Theta$, with $\Theta$ (depicted by the black region) the set of mean marginal willingness' to pay for the value of visible commodities that rationalize the respondents' choices
5.2 Heterogeneity in diamondness across goods

Testing specifications of diamondness. The exercises in Subsection 5.1 restrict attention to interpersonal heterogeneity in preferences for value. It was shown that individuals are quite different with respect to their willingness to pay for value. For policy makers, however, it may be more interesting to study heterogeneity in the diamondness across commodities. This heterogeneity has important consequences for optimal indirect taxation, as shown by Ng (1987).

On a more technical level, the analyses in Subsection 5.1 focus on the recovery of unknown elements of the decision-making process, such as the diamondness $\theta^n$ and the marginal willingness to pay for value $\theta^n_t$. In the current subsection, the focus is on testing different specifications of diamondness, i.e. performing the tests from Proposition 2 conditional on the diamondness vector. These tests can account for both the goodness-of-fit (i.e. pass rates) and the permissiveness (i.e. low power) of the specifications. On the one hand, the correct specification of diamondness is expected to lead to higher pass rates and hence more convincing models. On the other hand, increasing diamondness also leads to more permissive models. For this reason, information on the goodness-of-fit of the specifications is complemented with information on discriminatory power, which measures a model's ability to reject consistency of random, simulated datasets. Furthermore, the empirical performance is assessed at the individual level, thereby accounting for possible variation in rationality.

Table 1 presents the pass rates associated with different specifications of the model. The rows represent different degrees of diamondness associated with food at home and fuel, while the columns represent different degrees of diamondness associated with food away from home and clothing/luxuries. Remember that when one particular $\theta^n$ equals 0, the respective commodity is valued for its intrinsic consumption component only. By contrast, when $\theta^n$ equals 1, any combination of shadow prices $\theta^n_t$ is possible (including the scenario where $n$ is a pure diamond good).

The upper left result in Table 1 is the pass rate of the standard model (i.e. without preferences for value). The behavior of about 56 per cent of consumers can be ratio-
nalized by a well-behaved utility function of the form $U(Q)$. The lower right result corresponds to the model in which all commodities may be valued for their value. Not surprisingly, this revealed preference model imposes no testable restrictions, as a result of which all datasets are rationalized. The other cells are more interesting. By varying the relevant parameters, very different pass rates are obtained.

Power estimates (using the bootstrap approach) are presented in Table 2. The bootstrap measure simulates, for each observation of each respondent, budget shares from the distribution of observed shares across the sample. Subsequently, the selected—random—budget shares are multiplied by the consumer’s budget and divided by the prices associated with the observation under consideration. In this way, 8,200 random datasets (82 respondents times 100 iterations) are constructed. Discriminatory power then equals one minus the average pass rate of the random datasets. The more powerful specifications of the model are less permissive and therefore, are desirable. The power estimates in Table 2 are decreasing in function of the level of diamondness.

Next, the predictive success results in Table 2 summarize power and the pass rates from Table 1, thereby indicating which specifications are empirically supported. The predictive success of the standard model amounts to 0.042. Increasing the diamondness associated with visible consumption generally improves the predictive success results, whereas increasing the diamondness associated with less visible consumption lowers the predictive success scores. In particular, the highest predictive success is obtained when visible commodities have full diamondness and the less visible commodities have no diamondness. The corresponding predictive success more than doubles the GARP result. Specifically, predictive success has increased to 0.090. Admittedly, this rejects only 9 percent more of the random datasets compared to the actual observed datasets. The question is whether this difference in predictive success is statistically significant. Demuynck (2015) sets out a procedure to construct confidence bounds around mean predictive success scores, based on information on pass rates and power estimates at the individual level. The results are in Table 2. On the one hand, neither the predictive success of the standard model nor the predictive success of specifications that treat less visible goods as diamond goods are statisti-
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<tr>
<td></td>
<td>0</td>
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<tr>
<td>less visible goods</td>
<td>0</td>
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<td></td>
<td>0.25</td>
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<td></td>
<td>0.5</td>
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<td></td>
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Table 1: Pass rates (less visible goods=fuel and food at home; visible goods=food away from home and clothing/luxuries)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>metastasis</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
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<tbody>
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Table 2: Bootstrap power and predictive success with confidence intervals [lower 90%—confidence bound, upper 90%—confidence bound]: less visible goods = {fuel, food at home}, visible goods = {food away from home, clothing/luxuries}
cally different from 0 at the 10 per cent level. On the other hand, the mean predictive success associated with specifications that treat visible goods as pure diamond goods, appears statistically different from 0. Intuitively, this means that the modified model can describe the observed decisions while it rejects most of the random behavior.

The bad performance of specifications in the lower left region of Table 2, which attribute high levels of diamondness to less visible commodities, is mainly due to the power adjustment. Increasing the diamondness of these commodities from 0 to 1 rationalizes 41.9 percent more of the random datasets. This raises the question whether the results are robust to alternative power measures. Table 3 reports predictive success scores based on Bronars (1987)'s power measure. Bronars proposed an alternative hypothesis based on random draws of budget shares from a uniform distribution on the unit interval, rather than the empirical distribution of shares. In this setting, Bronars (1987)'s procedure is slightly modified to take the large number of zero expenditure items in the data into account\(^{13}\). Specifically, the modified method first computes–per commodity and per household–the proportion of strictly positive expenditures across the 11 observations. Subsequently, if a simulated budget share is above this threshold, the corresponding budget share is set to zero. Otherwise, a new budget share is drawn. This procedure mimics the (true) percentage of zeroes in the data (whereas the standard simulation approach has a zero probability of generating zeroes). Yet, unlike the bootstrap power index, the simulated shares do not necessarily correspond to an actually observed set of shares in some observation of some household. The results from Table 2 are confirmed in Table 3. First, predictive success scores generally increase in function of the diamondness of visible goods. None of the predictive success scores is statistically different from 0 at the 10 per cent level.

The results in Tables 2 and 3 are mean predictive success scores in the sample, but the revealed preference approach also provides us with information on the predictive success at the individual level. Indeed, each economic agent is characterized by a unique level of predictive success\(^{14}\). Table 4 presents the minimum, median, and

\(^{13}\) Cherchye et al. (2009) apply this procedure to a similar sample from the RLMS.

\(^{14}\) Individual predictive success is the difference between a binary pass/fail indicator and the pass
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Table 3: Bronars’ power and predictive success with confidence intervals [lower 90%—confidence bound, upper 90%—confidence bound]: less visible goods = {fuel, food at home}, visible goods = {food away from home, clothing/luxuries}
maximum predictive success across all individuals in the sample, as well as quintiles of the distribution. The standard model is compared to the specification with full diamondness for the visible commodities only.

The median predictive success is always positive. This implies that the observed choices of at least 50 per cent of the consumers in our sample can be described and can be empirically distinguished from random behavior. Furthermore, the median of the alternative model is somewhat higher. More importantly, the second quintile of the predictive success distribution is positive for the alternative model. Thus, the alternative model provides an accurate description of at least 60 per cent of the respondents, at the expense of losing some precision for individuals at the upper end of the predictive success distribution.

**Diamondness and Heffetz (2011)’s visibility index.** The results in Tables 2 and 3 indicate that the more visible commodities in particular are diamond goods. This argument is investigated more thoroughly by considering subclasses of the goods (food at home, food away from home, clothing/luxuries, and fuel) and by linking their diamondness to the corresponding visibility score by Heffetz (2011). The aim of this exercise is to provide some intuition for the positive relationship between diamondness and visibility. As indicated in the introduction, the presented methodology in principle can pick up price-dependent preferences from various sources. Preferences can depend on prices because of quality effects and status/diamond effects. It would be expected that the diamondness measure in this application picks up status/diamond effects in particular. After all, the aggregate commodities under consideration are heterogeneous in terms of visibility to society rather than heterogeneous in terms of quality.

First, all commodities are ordered according to Heffetz’ visibility index. This gives the following ranking: (1) clothing/luxuries, (2) food away from home, (3) food at home, and (4) fuel. The question is whether an alternative ranking based on diamondness follows this ordering. Figure 7 sets out the predictive success in function of the diamondness of each subclass of commodities. Starting from the rate of random datasets, simulated using the bootstrap procedure.
specification that attributes no diamondness to less visible goods and diamondness of one to more visible goods—this corresponds to the specification with the highest predictive success in Tables 2 and 3—the diamondness of each commodity is varied from 0 to 1, ceteris paribus. The impact on predictive success is presented in Figure 7.

It turns out that predictive success is gradually increasing in the diamondness of food consumed away from home. The predictive success associated with maximum diamondness is 3.3 percentage points higher than the predictive success associated with no diamondness. Similarly, the empirical performance of the model is increasing in the diamondness of clothing and luxuries, although the relationship is not monotone. Predictive success is improved by 1.5 percentage points. On the other hand, for fuel, the predictive success peaks at a diamondness level around 0.4, but a diamondness of 1 is slightly worse than a diamondness of 0. Furthermore, the predictive success peaks when the diamondness of food at home is 0, that is, when food at home is modeled as a standard good. Finally, Figure 7 also presents 90% confidence bounds around the predictive success estimates. The lower bounds associated with visible goods are slightly below 0 at lower diamondness levels and slightly above 0 at higher diamondness levels, indicating a significant improvement in predictive success. However, the differences are very small in absolute terms.

Ranking these commodities based on predictive success gains gives the following result: (1) food away from home, (2) clothing/luxuries, (3) fuel, and (4) food at home. Clearly, this result ranks the more visible commodities “food away from home” and “clothing/luxuries” higher than the less visible goods “fuel” and “food at home.” The positive relationship between visibility and diamondness is confirmed. On the other hand, the diamondness of food away from home—as opposed to the diamondness of clothing and luxuries—is most pronounced.

This raises the question whether the application also picks up quality effects within and across commodities. The answer to the first question—does the application pick up effects of the quality composition within aggregates?—depends, in

15The gain captures the difference between the predictive success when \( \theta \) is 1 and the predictive success when \( \theta \) is 0.
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Table 4: Distribution of individual predictive success scores in the sample with $\theta = [\theta^1 = \text{diamondness of less visible commodities}, \theta^2 = \text{diamondness of more visible commodities}]$

Figure 7: Predictive success in function of diamondness per commodity (with predictive success gains = predictive success when $\theta$ is 1 − predictive success when $\theta$ is 0) and the corresponding 90% confidence bounds.
part, on the sensitivity of an aggregate to changes in its composition. In the application under consideration, the price of an aggregate is independent of changes in the underlying quality composition. Heterogeneity in quality within aggregates is therefore not automatically incorporated in the aggregate prices. Furthermore, quality variation that affects the aggregate quantities is expected to weaken the link between visibility and diamondness. After all, there is no reason to assume that quality considerations are less important for food at home and fuel. Yet these less visible commodities turn out to have lower diamondness. The latter question—does the application pick up quality effects across commodities?—is mitigated by the fact that the commodities in the current application are aggregates. Although consumers may judge the quality of similar goods by price (Scitovsky (1945)), it seems less natural to infer the quality of distinct commodity groups (e.g., food, fuel, and jewelry) from relative price variation. Finally, the high diamondness of food away from home is consistent with findings in the literature on the positionality of commodities (see, e.g., Solnick and Hemenway (1998, 2005) and Carlsson et al. (2007)). In this research, food away from home is typically characterized as a highly positional commodity. In fact, the high diamondness of food away from home partly explains why the corresponding budget share is less sensitive to price variation in Figure 3 compared to the budget share spent on clothing.

6  Conclusion

This study extends the revealed preference approach to deal with preferences for value. Preferences for value, as a special case of price-dependent preferences, imply that consumers derive utility from the value of goods, and not only from the consumed quantity. Identification of preferences for value (i.e., diamondness) is important for the design of optimal indirect tax schedules, but it is difficult to distinguish the value-dependent preferences from budget-constraint price effects. In this study, diamondness is defined as a limit on the marginal willingness to pay for value associated with commodities.
Ng (1993)’s model is extended in the sense that multiple commodities can trigger preferences for value, and diamondness can take any value from 0 to 1. A diamondness of 0 corresponds to standard goods whereas a diamondness of 1 allows for pure diamond goods. In this sense, the presented approach fits in the “PEEMs” research agenda supported by Rabin (2013), which aims to develop tractable refinements of existing models.

When value enters the utility function, it is still possible to formulate testable conditions even if no parametric structure is imposed on the utility functions. Furthermore, the different levels of marginal willingness to pay for value are generally nonnested, meaning that set-identification is possible.

Finally, this methodology is applied to a data sample from the RLMS. Given this set-up, finding preferences for value driven by conspicuous consumption (status effects) may be expected, because the commodities under consideration are heterogeneous in terms of their visibility to society. This is the first application of revealed preference that explicitly incorporates preferences for value.

The results indicate that the predictive success of a standard GARP test is significantly below the predictive success of alternative specifications that set strictly positive marginal willingness to pay for value. The diamondness measure is positively related to Heffetz (2011)’s visibility index. Interestingly, treating more visible commodities, such as food away from home and clothing/luxuries, as diamond goods improves the predictive success of the consumer behavior models. Following the argument of Ng (1987), special taxation rules may be appropriate.

There are different avenues for further research. First, although the current results are already interesting, a larger sample may shed more light on the variation in diamondness across consumers. Second, the presented revealed preference tests apply to unitary decisions, for example, by single consumers. In a collective setting, variation in intra-household bargaining power would need to be taken into account, too. This further complicates the analysis, for prices affect consumption decisions in three ways: through the budget constraint, through possible variation in intra-household bargaining power, and because of individual preferences for value. Finally, this study formulates a revealed preference framework for the analysis of
value-dependent preferences. This opens the door for many new applications of this nonparametric methodology, which go beyond the application presented in this study. Future research could apply this methodology to, for instance, experimental data. By varying the visibility of the commodities in the experiment, pure diamond effects could be distinguished from status (conspicuous consumption) effects. By varying the level of available information and the degree of similarity between commodities, diamond/status effects could be distinguished from quality effects. After all, judging quality by price seems all the more relevant when products are similar and information is limited.

References


A Proof of Proposition 2

Proof. This proof shows the equivalence of Statements 1 and 2. Equivalence between Statements 2 and 3 follows from Varian (1982).
• Statement 1 implies Statement 2. Consider the following (necessary) first-order condition for the optimization of $OPT - \theta$:

$$\frac{\partial U}{\partial Q_n^t} + \frac{\partial U}{\partial M_n^t} \cdot P_n^t \leq \lambda_t P_n^t;$$

where $\frac{\partial U}{\partial Q_n^t}$ and $\frac{\partial U}{\partial M_n^t}$ are subderivatives of the concave utility function with respect to $Q_n^t$ and $M_n^t$, respectively. The inequalities are replaced with equalities if the quantities $Q_n^t$ are strictly positive.

Moreover, concavity of the utility function yields:

$$u_t - u_v \leq \sum_{n=1}^{[N]} \frac{\partial U}{\partial Q_v^n} \cdot (Q_n^t - Q_v^n) + \sum_{n=1}^{[N]} \frac{\partial U}{\partial M_v^n} \cdot (P_v^n Q_n^t - P_v^n Q_v^n); \quad (4)$$

Finally, replace $\theta_v^n = \frac{\partial U}{\partial M_v^n} \lambda_v$ and $\pi_v^n = (1 - \theta_v^n) P_v^n \geq \frac{\partial U}{\partial Q_v^n} \lambda_v$. For strictly positive quantities $Q^n$, $\pi_v^n = \frac{\partial U}{\partial Q_v^n} \lambda_v$. Thus, consistency with condition (4) requires consistency with condition (5).

$$u_t - u_v \leq \lambda_v \sum_{n=1}^{[N]} \pi_v^n (Q_n^t - Q_v^n) + \lambda_v \sum_{n=1}^{[N]} \theta_v^n (P_v^n Q_n^t - P_v^n Q_v^n); \quad (5)$$

This concludes the necessity part.

• Statement 2 implies Statement 1, based on Varian (1982).

Start from the observation that

$$U(Q, M) \leq U_v + \lambda_v \sum_{n=1}^{[N]} \pi_v^n (Q^n - Q_v^n) + \lambda_v \sum_{n=1}^{[N]} \theta_v^n (P_v^n Q^n - P_v^n Q_v^n);$$

In the following step, select the minimum of all overestimates:

$$U(Q, M) = \min_v (U_v + \lambda_v \sum_{n=1}^{[N]} \pi_v^n (Q^n - Q_v^n) + \lambda_v \sum_{n=1}^{[N]} \theta_v^n (P_v^n Q^n - P_v^n Q_v^n));$$
This formulation should be such that any \( (Q, M) \) for which \( P_t'Q \geq P_t'Q \) and 
\[ M = P_t \odot Q, \]
implies that \( U(Q_t, M_t) \geq U(Q, M) \).

First, it is important to understand that 
\[
U(Q_v, M_v) = U_v \quad \text{for} \quad v \in T.
\]
Indeed, for some \( t \),
\[
U(Q_v, M_v) = U_t + \lambda_t \sum_{n=1}^{|N|} \pi^n_t(Q_v^n - Q^n_t) + \lambda_t \sum_{n=1}^{|N|} \theta^n_t(P^n_t Q^n_v - P^n_t Q^n_t)
\leq U_v + \lambda_v \sum_{n=1}^{|N|} \pi^n_v(Q_v^n - Q^n_v) + \lambda_v \sum_{n=1}^{|N|} \theta^n_v(P^n_v Q^n_v - P^n_v Q^n_v)
= U_v
\]
If this inequality were strict,
\[
U_v - U_t > \lambda_t \sum_{n=1}^{|N|} \pi^n_t(Q_v^n - Q^n_t) + \lambda_t \sum_{n=1}^{|N|} \theta^n_t(P^n_t Q^n_v - P^n_t Q^n_t)
\]
which contradicts the Afriat inequalities. Hence, \( U(Q_v, M_v) = U_v \).

Second, any \( (Q, M) \) for which \( P_t'Q \geq P_t'Q \) and 
\[ M = P_t \odot Q, \]
must be consistent with
\[
U(Q, M) = \min_v (U_v + \lambda_v \sum_{n=1}^{|N|} \pi^n_v(Q^n - Q^n_v) + \lambda_v \sum_{n=1}^{|N|} \theta^n_v(P^n Q^n_v - P^n Q^n_v))
\leq U_t + \lambda_t \sum_{n=1}^{|N|} \pi^n_t(Q^n - Q^n_t) + \lambda_t \sum_{n=1}^{|N|} \theta^n_t(P^n Q^n_t - P^n Q^n_t)
\leq U_t = U(Q_t, M_t)
\]
The first inequality follows from the definition of \( U(Q, M) \), the second in-
equality follows from

\[
U_t + \lambda_t \sum_{n=1}^{\lvert N \rvert} \pi_t^n (Q^n - Q_t^n) + \lambda_t \sum_{n=1}^{\lvert N \rvert} \theta_t^n (P_t^n Q^n - P_t^n Q_t^n)
\]

\[
= U_t + \lambda_t \sum_{n=1}^{\lvert N \rvert} (1 - \theta_t^n)(P_t^n Q^n - P_t^n Q_t^n) + \lambda_t \sum_{n=1}^{\lvert N \rvert} \theta_t^n (P_t^n Q^n - P_t^n Q_t^n)
\]

\[
= U_t + \lambda_t \sum_{n=1}^{\lvert N \rvert} P_t^n (Q^n - Q_t^n)
\]

\[
\leq U_t
\]

which uses that \( \pi_t^n = (1 - \theta_t^n)P_t^n \). This concludes the sufficiency part.

B  Hicks’ composite commodity theorem in the diamond setting

This proof shows that Hicks’ composite commodity theorem also applies in a setting with diamond goods. Otherwise stated, the revealed preference tests shed light on the consistency of observed consumption patterns with specifications of \( \theta \), regardless of whether Proposition 2 is applied to the aggregates or to the goods. However, this is conditional on fixed relative prices and diamondness within aggregates.

**Proof.** Start from aggregate prices, which are constructed in the following way

\[
P_v^n = (p_l^v)^w (p_h^v)^{1-w}
\]

and from the assumption that there is no relative price variation within subgroups of products, that is, \( \frac{p_h^v}{p_l^v} = \alpha \).

- First, based on this information, \( Q_v, p_l^v, \) and \( p_h^v \) can be expressed in terms of constants \( \beta_l = \alpha^{w-1} \) and \( \beta_h = \alpha^w \), which are invariant across periods \( v \).
\[ Q_v = \beta^l q^l_v + \beta^h q^h_v, \quad p^l_v = \beta^l P_v \text{ and } p^h_v = \beta^h P_v \]

In order to observe this, the following are used: \( \frac{p^h_v}{p^l_v} = \alpha \) and \( P^n_v = (p^l_v)^w (p^h_v)^{1-w} : \)

\[
Q_v = \frac{p^l_v q^l_v + p^h_v q^h_v}{P^n_v} = \frac{p^l_v q^l_v + p^h_v q^h_v}{(p^l_v)^w (p^h_v)^{1-w}} = \frac{p^l_v q^l_v + \alpha p^l_v q^h_v}{(p^l_v)^w (\alpha p^h_v)^{1-w}} \\
= \frac{p^l_v (q^l_v + \alpha q^h_v)}{p^l_v (\alpha^{1-w})} = \frac{q^l_v + \alpha q^h_v}{\alpha^{1-w}} = \alpha \frac{q^l_v}{\alpha^{1-w}} + \alpha^w q^h_v = \beta^l q^l_v + \beta^h q^h_v
\]

\[
P_v = (p^l_v)^w (p^h_v)^{1-w} = (p^l_v)^w (\alpha p^h_v)^{1-w} = \alpha^{1-w} p^l_v \\
\Rightarrow p^l_v = \beta^l P_v
\]

\[
P_v = (p^l_v)^w (p^h_v)^{1-w} = (\frac{p^h_v}{\alpha})^w (p^h_v)^{1-w} = \alpha^{-w} P_v \\
\Rightarrow p^h_v = \beta^h P_v
\]

- Second, assume that \( \theta^n_v = \theta^l_v = \theta^h_v \). Show the equivalence between the inequalities in Proposition 2 (Statement 2) applied to the subproducts and the inequalities in Proposition 2 (Statement 2) applied to the aggregates.

\[
u_t - u_v \leq \lambda_v (1 - \theta^1_v) p^{h1}_v (q^{h1}_t - q^{h1}_v) + \lambda_v (1 - \theta^2_v) p^{h2}_v (q^{h2}_t - q^{h2}_v) \\
+ \lambda_v (1 - \theta^1_v) p^{l1}_v (q^{l1}_t - q^{l1}_v) + \lambda_v (1 - \theta^2_v) p^{l2}_v (q^{l2}_t - q^{l2}_v) \\
+ \lambda_v \theta^1_v (p^{h1}_v q^{h1}_t - q^{h1}_v q^{h1}_v) + \lambda_v \theta^2_v (p^{h2}_v q^{h2}_t - q^{h2}_v q^{h2}_v) \\
+ \lambda_v \theta^1_v (p^{l1}_v q^{l1}_t - q^{l1}_v q^{l1}_v) + \lambda_v \theta^2_v (p^{l2}_v q^{l2}_t - q^{l2}_v q^{l2}_v)
\]
\[\Leftrightarrow u_t - u_v \leq \lambda_v (1 - \theta_v^1) P_v^1 (\beta^{h1} q_t^{h1} - \beta^{l1} q_t^{l1}) + \lambda_v (1 - \theta_v^2) P_v^2 (\beta^{h2} q_t^{h2} - \beta^{l2} q_t^{l2})
\]
\[+ \lambda_v (1 - \theta_v^1) P_v^1 (\beta^{l1} q_t^{l1} - \beta^{h1} q_v^{h1}) + \lambda_v (1 - \theta_v^2) P_v^2 (\beta^{l2} q_t^{l2} - \beta^{h2} q_v^{h2})
\]
\[+ \lambda_v \theta_v^1 (P_t^1 \beta^{h1} q_t^{h1} - P_v^1 \beta^{h1} q_v^{h1}) + \lambda_v \theta_v^2 (P_t^2 \beta^{h2} q_t^{h2} - P_v^2 \beta^{h2} q_v^{h2})
\]
\[+ \lambda_v \theta_v^1 (P_t^1 \beta^{l1} q_t^{l1} - P_v^1 \beta^{h1} q_v^{h1}) + \lambda_v \theta_v^2 (P_t^2 \beta^{l2} q_t^{l2} - P_v^2 \beta^{h2} q_v^{h2})
\]

\[\Leftrightarrow u_t - u_v \leq \lambda_v (1 - \theta_v^1) P_v^1 (\beta^{h1} q_t^{h1} + \beta^{l1} q_t^{l1} - \beta^{h1} q_v^{h1} - \beta^{l1} q_v^{l1})
\]
\[+ \lambda_v (1 - \theta_v^2) P_v^2 (\beta^{h2} q_t^{h2} + \beta^{l2} q_t^{l2} - \beta^{h2} q_v^{h2} - \beta^{l2} q_v^{l2})
\]
\[+ \lambda_v \theta_v^1 (P_t^1 \beta^{h1} q_t^{h1} + P_t^1 \beta^{l1} q_t^{l1} - P_v^1 \beta^{h1} q_v^{h1} - P_v^1 \beta^{l1} q_v^{l1})
\]
\[+ \lambda_v \theta_v^2 (P_t^2 \beta^{h2} q_t^{h2} + P_t^2 \beta^{l2} q_t^{l2} - P_v^2 \beta^{h2} q_v^{h2} - P_v^2 \beta^{l2} q_v^{l2})
\]

\[\Leftrightarrow u_t - u_v \leq \lambda_v (1 - \theta_v^1) P_v^1 (Q_t^1 - Q_v^1) + \lambda_v (1 - \theta_v^2) P_v^2 (Q_t^2 - Q_v^2)
\]
\[+ \lambda_v \theta_v^1 (P_t^1 Q_t^1 - P_v^1 Q_v^1) + \lambda_v \theta_v^2 (P_t^2 Q_t^2 - P_v^2 Q_v^2)
\]