Optimal Structure of Fiscal and Monetary Authorities

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Abstract
Advanced economies pair an elected fiscal authority with an independent monetary authority. This paper illustrates why monetary and fiscal authorities are designed so that monetary independence is beneficial. Replicating the advanced economies’ structure with authorities microfounded by a political economy model shows that this structure is the solution to a constrained mechanism design problem that overcomes time inconsistency and results in the highest possible welfare. Goal and instrument independence, singly and in combination, are insufficient to minimize time inconsistency, though their combination is necessary.

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1 Introduction

Fischer (1994) describes two forms of independence for a monetary authority: goal independence – a monetary authority may have different goals than the fiscal authority has – and instrument independence – a monetary authority may have different instruments than the fiscal authority does to achieve its goals. Rather than taking the existing structure of monetary and fiscal authorities as given, this paper analyzes why monetary and fiscal authorities are designed such that an independent monetary authority is beneficial. The two types of independence are explored in a macro model featuring political economy microfoundations from Battaglini and Coate (2008). Goal and instrument independence, both individually and combined, are necessary but not sufficient to eliminate the time inconsistency problem of nominal debt, as in Kydland and Prescott (1977) and Barro and Gordon (1983). The unique structure of monetary and fiscal authorities that ameliorates time inconsistency features an elected fiscal authority and a goal and instrument independent monetary authority. This structure results in the highest possible welfare by allowing the use of nominal bonds for tax smoothing.

A fiscal authority controls distortionary taxation and nominal bonds while an instrument independent monetary authority controls the price level. Separating authorities separates instruments – and possibly goals – in an attempt to solve the time inconsistency problem of nominal debt: a benevolent monetary authority will inflate away the real value of nominal debt, equivalent to consumer wealth, at the start of every period, allowing a fiscal authority to set taxes to the minimum. Consumers anticipate this inflation and will not hold nominal bonds whose real value will disappear.

There are four structures for analyzing fiscal and monetary authority design: each authority can be benevolent, with a goal is to maximize the welfare of all citizens, or elected in the manner of Battaglini and Coate (2008), with a goal to maximize only the welfare of citizens who voted for it. The structures are illustrated in Figure 1.

The structure with an elected fiscal authority and a benevolent monetary
authority is unique in minimizing time inconsistency and allowing the use of nominal bonds. Choosing this structure can be viewed as a type of mechanism or constitutional design problem: citizens in period 0 would select this structure before knowing whether or not they are members of the governing coalition in period 1. While this structure features goal and instrument independence, they are not sufficient to maximize welfare; the specific structure is important. The structure with a benevolent fiscal authority and an elected monetary authority also features goal and instrument independence, but fails to overcome time inconsistency.

Electing the fiscal authority while keeping the monetary authority benevolent, thus making the two authorities goal and instrument independent, leads to the highest welfare. This result, a byproduct of Miller (2016), comes from the pairing partially solving the time inconsistency problem of nominal debt: if the benevolent monetary authority maintains a positive level of nominal debt, the debt will constrain wasteful spending by the elected fiscal authority while allowing some tax smoothing. The welfare benefit of electing the fiscal authority is unique; electing the monetary authority does not have any benefit.

Electing the monetary authority while keeping the fiscal authority benev-
olent, despite featuring goal and instrument independence, does not alleviate time inconsistency. An elected monetary authority knows that inflating away the real value of the bonds held by everyone will eliminate the real value of bonds held by the subset of citizens who elected it. The logic of time inconsistency is identical to the classical case where both authorities are benevolent. The elected monetary authority will inflate, eliminating the real wealth of its coalition, in order to have the fiscal authority set distortionary taxes to their lowest level.

Electing both the fiscal and monetary authority (or equivalently, when fiscal authority captures the monetary authority) also results in time inconsistency, but for new reasons. The elected monetary authority will inflate, eliminating the real wealth of its coalition and everyone else, in order to have the fiscal authority direct transfers back to the coalition. For members of the coalition, the benefit from the transfers the fiscal authority provides will be greater than the loss of real wealth due to inflation. Citizens who are not members of the coalition will lose their wealth without compensating transfers.

The main contribution of this paper is to refine ideas about monetary independence using explicit microfoundations to evaluate possible structures for monetary and fiscal authorities. Many papers, most famously Rogoff (1985) and the branch of literature following it, use an independent, conservative central banker to limit consumers’ inflation expectations while choosing an output and inflation tradeoff. Modern forms of this analysis can be found in Adam and Billi (2008), Adam (2011), and Martin (2011). These papers include separate and independent fiscal and monetary authorities to examine the benefits of monetary conservatism. This paper’s political economy microfoundations provide a way to analyze why we utilize a structure where independence matters. Rather than feature a conservative central banker, this paper, splits goals between the monetary and fiscal authority through elections.

Niemann et al. (2013) examines how a monetary authority can exert some control over a fiscal authority through the use of different monetary instruments. There is a tradeoff between time inconsistency and enhancing fiscal power. In this paper there is no tradeoff, the monetary authority benefits
from a distorted fiscal authority. The goal independence of the fiscal and monetary authorities relies on political economy rather than myopia.

Romer and Romer (1997) analyzes different attempts to design a monetary authority that will produce stable inflation. Persson and Tabellini (1993) and Walsh (1995) model the interaction between monetary and fiscal authorities in a principal agent setup. They attempt to design optimal contracts to control the monetary authority. The structures analyzed in this paper do not rely on optimal contracting. Monetary control, or lack thereof, arises endogenously from the design of the structure itself.

Empirically, papers such as Alesina and Summers (1993), Cukierman et al. (1992) and Carlstrom and Fuerst (2009) link measures of central bank independence with performance on inflation metrics. There are many complications to forming good measures of central bank independence. Even the causality of independence leading to low inflation has been questioned in Posen (1998). This paper shows theoretically that independence itself is not the cause of inflation control: independence combined with specific structural details can jointly lead to lower inflation.

The rest of the paper analyzes the four different structures. Section 2 explains the model. Section 3 shows that of the four possible structures, only a benevolent monetary authority and elected fiscal authority is able to sustain bonds. Finally, Section 4 concludes.

2 The Model

Nominal government debt, when sustainable, links periods. Fiscal policy consists of setting taxes, expenditure on a public good, direct transfers to citizens, and nominal bond issuance. The timing in a period is as follows: a real shock determines wages (and the distortion due to taxes) at the beginning of every period. After the shock, the monetary authority sets the price level then the fiscal authority chooses its policy. When monetary and fiscal authorities are controlled by the same group, the timing of choices is immaterial. Figure 2 illustrates the sequence of decisions by authorities in a period.
2.1 Consumers

There are $n$ identical consumers, indexed by $i$ when necessary. A consumer’s per period utility function is

$$u(c, g, l) = c + A \log(g) - \frac{l^{1+1/\epsilon}}{\epsilon + 1}$$

and an individual seeks to maximize $U = \sum_{t} \beta^t u(c_t, g_t, l_t)$ where $c$ is a consumption good, $g$ is government spending on a public good, $l$ is labor, and $\beta$ the discount rate. The parameter $\epsilon > 0$ is the Frisch elasticity of labor supply. $A$ is a parameter allowing adjustment of the utility of government spending on the public good. Utility linear in consumption is used to eliminate wealth effects to preserve consumers homogeneity for the political process and simplify the interest rate in order to concentrate on the model’s main political mechanism.

A representative consumer $i$ in period $t$ faces the budget constraint

$$c + qB'_i \leq w_{\theta}l(1 - \tau) + \frac{B_i}{P(B)} + T_i$$

Figure 2: Timing of Monetary and Fiscal Decisions
Variables without a prime refer to variables in period $t$ while variables with a prime refer to variables in period $t + 1$. The consumer can consume $c$ or purchase nominal bonds $B_i'$ at a price $q$ where each bond pays a nominal unit of income in the next period. $B = \sum_i B_i$ is the total number of bonds in the economy$^1$.

The consumer’s income consists of labor income at wage $w_\theta$ that is taxed by the government at the distortionary tax rate $0 \leq \tau \leq 1$ together with direct transfers $T_i \geq 0$ from the government. $P(B)$ is the price level determined by the monetary authority at the start of the period as a function of the number of bonds.

The price level in the current period will generally be abbreviated as $P = P(B)$ and the price level in the next period as $P' = P(B')$. The price level is not an intertemporal variable. The ratio of current to next period price level $\frac{P}{P'}$ determines the real return on bonds. For simplicity I normalize this ratio by making bonds pay 1 nominal unit of income in the next period. Thus the real value of bonds will only depend on $P'$ which is set independently every period.

The consumer’s budget constraint and linear utility imply the equilibrium bond price

$$q(B') = \beta E_{\theta'} \left[ \frac{1}{P'(B')} \right]$$

where the expectation is over possible realizations of the wage shock in the next period.

A consumer’s utility is defined entirely by the government’s choices of taxation $\tau$ and public good spending $g$. Deriving the optimal amount of labor as a function of the tax rate $\tau$ shows

$$l_0^*(\tau) = (\epsilon w_\theta (1 - \tau))^\epsilon$$

Plugging this into the consumer’s utility function shows the indirect utility

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$^1$With identical consumers, the symmetric equilibrium examined in the paper induces all consumers to hold identical bonds $B_i = \frac{B}{n}$. The choices of fiscal and monetary authorities will depend only on aggregate bonds.
function before transfers is

\[ W_\theta(\tau, g) = \frac{\epsilon^\epsilon (w_\theta (1 - \tau))^{\epsilon + 1}}{\epsilon + 1} + A \log(g) \]  

(5)

2.2 Firms

The representative firm has a linear production technology

\[ z = w_\theta l \]  

(6)

used to produce an intermediate good \( z \) at wage \( w_\theta \) with labor \( l \). At the beginning of each period an i.i.d. technology shock hits the economy such that wages \( w_\theta \in \{ w_l, w_h \} \) where \( w_l < w_h \). The probability that \( w_\theta = w_h \) is \( \pi \), the probability that \( w_\theta = w_l \) is \( 1 - \pi \).

The intermediate good \( z \) is split costlessly between the consumption good \( c \) and the public good \( g \) such that

\[ c + g = z. \]  

(7)

This defines the per period resource constraint

\[ c + g = w_\theta l. \]  

(8)

2.3 Government

The government controls fiscal policy. Raising revenue is possible via a distortionary labor tax \( \tau \) and by selling nominal bonds \( B' \). A positive bond level means the government is in debt hence owes revenue to consumers. The government must repay nominal bonds \( \frac{B}{P} \). It can then spend leftover revenue on a public good \( g \) that benefits all \( n \) citizens or on non-negative transfer payments \( T_i \) that benefit individuals.
The government’s budget constraint is

\[ g + \sum_i T_i + \frac{B}{P} \leq \text{Rev}_\theta(\tau) + qB' \quad (9) \]

where

\[ \text{Rev}_\theta(\tau) = n\tau w_\theta(\epsilon w_\theta(1 - \tau)) \epsilon \quad (10) \]

is the total tax revenue raised by the distortionary labor tax on all \( n \) consumers.

Define the budget surplus before transfers as

\[ S_\theta(\tau, g, B'; B) = \text{Rev}_\theta(\tau) + qB' - g - \frac{B}{P}. \quad (11) \]

The surplus must be large enough to pay for any transfers hence \( S_\theta(\tau, g, B'; B) \geq \sum_i T_i \). Transfers themselves must be non-negative: \( \forall i \ T_i \geq 0 \).

There are endogenous limits to the amount of bonds the government can issue. The upper bound on debt is defined as the maximum amount of bonds the government is able to repay in the case of the bad realization of the wage shock \( w_l \) if it spends nothing on the public good and transfers. Define the upper bound \( B \) as

\[ B = \max_{\tau} \frac{\text{Rev}_\theta(\tau)}{q(\theta)}. \]

The lower bound on debt is the amount of bonds such that revenue from the bonds would be sufficient to fund optimal public good spending without utilizing the distortionary labor tax. The optimal amount of public good spending is \( g^* \) such that \( \frac{nA}{g^*} = 1 \). This equation equates the declining marginal benefit of providing the public good with the opportunity cost to consumers (which is consuming that revenue directly with linear utility). Define \( B \) as

\[ B = -\frac{nA}{q(\theta)} \]

the level of bonds where one more unit of government spending has the same marginal utility as individual consumption. This is the Samuelson level of bonds \( B \), the bond level where the government is entirely funded by interest from bonds, all future distortionary labor taxes are zero, and public good spending is constant at the optimum \( g^* \).
2.4 Monetary Authority

The monetary authority chooses the price level $P$. Inflation is costless. Both these modeling choices are made to emphasize the political mechanism at work in the model. Choosing $P$ directly is equivalent to the monetary authority controlling the interest rate on nominal bonds, as the monetary authority would do in the cashless limit via infinitely small open market operations that result in zero seignorage. Additionally, moving $P$ rather than operating through money holdings simplifies the model by allowing us to consider the simple indirect utility function with a correspondingly simple interest rate, and not split money and bond holdings.

When the two authorities are controlled by different groups, the model utilizes timing akin to a Stackelberg game with the monetary authority as leader and the government as follower. The monetary authority chooses the price level $P$ after the shock in each period. Since $P$ determines the payout to bonds, choosing $P$ is equivalent to choosing the interest rate on those bonds. Thus monetary policy controls the real value of government debt which is equivalent to consumer wealth. After the monetary authority moves, the fiscal authority chooses its fiscal instruments. When the two authorities are controlled by the same group, all instruments are chosen at the same time. See Figure 1. [Miller (2016)] contains an in-depth discussion of this timing in a similar model.

The monetary authority lacks commitment. Each period the monetary authority chooses the price level for that period only and cannot credibly promise what it will do in the future. Specifically, I constrain the monetary authority to choose the price level solely as a function of its information set \( \{B, w_0\} \) at the beginning of a period. This is a consequence of the monetary authority’s lack of commitment, and narrows the space of possible equilibria.

The monetary authority can only use current variables because strategies that threaten non-optimal ex-post actions, such as trigger strategies, require commitment to maintain the threat. The monetary authority is restricted to the current bond level and shock because bonds are the only intertemporal good and thus the only variable the monetary authority can observe at the
beginning of a period after realization of the shock. Predicating monetary
policy on fiscal policy decisions that took place in previous periods or will
take place in future periods is ruled out because it is a form of commitment.

2.5 Self-Interested Policy

We will analyze the consequences of motivating the monetary or fiscal au-
thority, or both, by a political process. A benevolent authority attempts to
maximize the welfare of all citizens. A self-interested authority attempts to
maximize the utility of a subgroup of the citizenry. In this section, I provide
an overview of the political equilibrium used in the model.

Following the political system laid out in [Battaglini and Coate (2008)] and
[Miller (2016)] who extend the political economy model of [Baron and Ferejohn
(1989)], citizens vote each period to decide that period’s policy. If the fiscal
authority is self-interested and the monetary authority benevolent, the policy
variables are \{τ, g, B, \{Ti\}_i^n\}. If the monetary authority is self-interested and
the fiscal authority benevolent, the policy variable is the price level \(P\). If
both the monetary and fiscal authority are self-interested, the policy variables
are \{P, τ, g, B, \{Ti\}_i^n\}. For simplicity, I require the coalition controlling the
monetary and fiscal authorities to be the same citizens.

In each period there are \(T\) rounds of voting to determine policy. Each
round of voting starts with one citizen being randomly assigned the power to
propose a choice of policy variables. The proposal is enacted if \(m \leq n\) citizens
vote for it. If enacted, this ends the voting for that period, a new round will
begin next period. If the proposal fails the voting round ends and a new round
begins with a new randomly selected proposer. There can be a maximum of
\(T\) proposal rounds after which a dictator is appointed. The dictator chooses
policies unilaterally with the constraint that all transfers \(T_i\) must be equal if
fiscal policy is considered.

A policy proposal defines the policy for a single period. The next period a
new proposer is randomly selected and the process begins anew. Commitment
across periods is impossible due to the design of the political system.
I focus on a symmetric Markov-perfect equilibrium for each structure. These are proposals that depend only on current productivity and debt \( \{w_\theta, B\} \) where \( \theta \in \{h, l\} \). The proposals are independent of both the history of the economy and proposal round. Thus we only need to examine the proposal in the first round.

In order for a proposal to be accepted, the proposal must make the members of the \( m \) coalition as well off as the expectation of waiting a round for the next proposal. In practical terms, proposers will propose instruments to maximize the utility of the \( m \) citizens in the coalition without care for non-coalition citizens. This is in contrast to the choices of a benevolent authority which will maximize the welfare of all \( n \) citizens.

3 Model Analysis

A benevolent authority attempts to maximize total welfare. Since consumers are identical and utility quasilinear, this is equivalent to maximizing the utility of a single average representative consumer. I analyze the four possible structures by grouping them by control of the fiscal authority. First, the analysis of the two structures with a benevolent fiscal authority shows that both are unable to support any bonds due to time inconsistency. Second, the analysis of the two structures with a self-interested fiscal authority shows that the structure with a self-interested monetary authority is unable to support bonds while the structure with a benevolent monetary authority can support a positive level of bonds.

3.1 The Self-Interested Authority’s Problem

To condense the description of very similar topics, I illustrate the problem of the self-interested authority in the two structures that will be analyzed: when only the monetary authority is self-interested and when both the fiscal and monetary authority are self-interested and share a coalition.

Following the outline of Barseghyan et al. (2013) and Miller (2016), I fo-
Focus on a symmetric Markov-perfect equilibrium for each potential structure. These are proposals that depend on current productivity and debt $\{w_\theta, B\}$. The proposals are independent of both the history of the economy and proposal round. A citizen will vote for a proposal if it makes him at least as well off as waiting for the next proposal round will. Hence a proposer will propose instruments that make citizens indifferent between voting for a proposal and waiting for the next round. I choose an equilibrium where proposals in each round are voted for by the necessary $m - 1$ citizens (and the proposer). This means that the equilibrium path consists of a single round with a single proposal that is voted for by the necessary citizens.

The equilibrium is a set of policy proposals for each round $r \in \{1, \ldots, T\}$ for the price level $\{P^r\}_1^T$ if the monetary authority is self-interested, fiscal instruments $\{\tau^r, g^r, B^{lr}, T^r_i\}_1^T$ if the fiscal authority is self-interested, or $\{P^r, \tau^r, g^r, B^{lr}, T^r_i\}_1^T$ if both are self interested. If the fiscal authority is self-interested, the transfers will be used by the proposer to convince a random group of $m - 1$ other citizens to support the proposal. Revenue not spent on transfers or public good spending is the effective transfer to the proposer. An equilibrium defines a value function $v^r_\theta(B)$ for each round representing the expected continuation payoff value for a citizen. The last value function $v^{T+1}_\theta(B)$ is the result of the default proposal by the dictator appointed after round $T$.

Given a set of value functions $\{v^r_\theta(B)\}_{r=1}^{T+1}$ the policy proposals must satisfy the proposer’s maximization problem. Similarly the policy proposals define the optimal value functions. I start with the first relationship. Since the first proposal in the first round is accepted, I drop the $r$ superscripts for simplicity. Formally, given the value functions, we can write the optimization problem for the policy proposals.

The three combinations of self-interested authorities will differ in the goal of the authority controlling each instrument. The model’s economy remains the same. For the structures where either authority is individually self-interested,
the optimization problem is

\[
\max_P \left[ \max_{\tau, g, B'} \left( W_{\theta}(\tau, g) + S_{\theta}(\tau, g, B'; \frac{B}{P}) - (m - 1)T_i + \beta \left[ \pi v_H(B') + (1 - \pi)v_L(B') \right] \right) \right]
\]

\[
\text{s.t.} \\
W_{\theta}(\tau, g) + T_i + \beta \left[ \pi v_H(B') + (1 - \pi)v_L(B') \right] \geq v^{r+1}_\theta(B) \\
T_i \geq 0 \forall i, S_{\theta}(\tau, g, B'; \frac{B}{P}) \geq (m - 1)T_i
\]

(12)

The constraints on fiscal policy exist whether fiscal choices are made by a benevolent or self-interested fiscal authority. The first constraint is the incentive compatibility constraint that states the proposal must make those citizens receiving a transfer at least as well off in expectation as they would be if they waited for the next proposal round. The other constraints force the proposal to be feasible given government’s budget constraint.

Similarly, for the structure where both monetary and fiscal authorities are self-interested, the optimization problem is

\[
\max_{P, \tau, g, B', T_i} \left[ W_{\theta}(\tau, g) + S_{\theta}(\tau, g, B'; \frac{B}{P}) - (m - 1)T_i + \beta \left[ \pi v_H(B') + (1 - \pi)v_L(B') \right] \right]
\]

\[
\text{s.t.} \\
W_{\theta}(\tau, g) + T_i + \beta \left[ \pi v_H(B') + (1 - \pi)v_L(B') \right] \geq v^{r+1}_\theta(B) \\
T_i \geq 0 \forall i, S_{\theta}(\tau, g, B'; \frac{B}{P}) \geq (m - 1)T_i
\]

(13)

Given policy choices, the value functions are determined by

\[
v_{\theta}(B) = \max_{\tau, g, B', T_i} \left[ W_{\theta}(\tau, g) + S_{\theta}(\tau, g, B'; \frac{B}{P}) \right]
\]

in the structure where only the monetary authority is self-interested,

\[
v_{\theta}(B) = \max_P \left[ W_{\theta}(\tau, g) + S_{\theta}(\tau, g, B'; \frac{B}{P}) \right]
\]

(14)

(15)
in the structure where only the fiscal authority is self-interested, and

\[
v_\theta(B) = W_\theta(\tau, g) + \frac{S_\theta(\tau, g, B', \frac{B'}{p})}{n} + \beta \left[ \pi v_H(B') + (1 - \pi) v_L(B') \right]
\] (16)
in the structure where both authorities are self-interested. These expressions come from the three possibilities for a citizen in a proposal round. In the structure where only the monetary authority is self-interested, the benvolent fiscal authority provides equal transfers to all citizens. In the structures where at least the fiscal authority is self-interested, with probability \( \frac{1}{n} \) a citizen is the proposer and thus receives the surplus after transfers. With probability \( \frac{m-1}{n} \) a citizen is not the proposer but is a member of the randomly selected coalition that votes for the proposal and thus receives the transfer \( T_i \). With probability \( \frac{n-m}{n} \) a citizen is not in the proposer’s coalition and receives no transfer. Since utility is quasilinear, the expected utility in a round is the payoff multiplied by the probability.

Because the first proposal is accepted in each round and all proposals will be identical, the value functions will be identical for every proposal round \( r \in \{1, \ldots, T\} \). If the round \( T \) proposal is rejected, the round \( T + 1 \) dictator’s proposal will result in the value function

\[
v_{\theta}^{T+1}(B) = \max_{P} \left[ \max_{\tau, g, B'} W_\theta(\tau, g) + \frac{S_\theta(\tau, g, B', \frac{B'}{p})}{n} + \beta \left[ \pi v_H(B') + (1 - \pi) v_L(B') \right] \right]
\] (17)

for the self-interested authority that could not come to an agreement in the structures where either authority is individually self-interested, and

\[
v_{\theta}^{T+1}(B) = \max_{P, \tau, g, B'} \left[ W_\theta(\tau, g) + \frac{S_\theta(\tau, g, B', \frac{B'}{p})}{n} + \beta \left[ \pi v_H(B') + (1 - \pi) v_L(B') \right] \right]
\] (18)
in the structure where both authorities are self-interested, subject to uniform transfers and the same feasibility constraints as before.

I characterize the equilibrium in each of the structures in the next subsections, and prove their existence in the appendix.
3.2 Structures with a Benevolent Fiscal Authority

The benevolent fiscal authority’s problem can be written as

$$\max_{\tau, g, B'} \left\{ \sum_{i} T_i \right\} \text{subject to}$$

$$W_\theta(\tau, g) + \frac{\sum_i T_i}{n} + \beta [\pi v_H (B') + (1 - \pi) v_L (B')]$$

The first constraint is that transfers must be non-negative and the surplus must be weakly positive. Any surplus will be distributed to citizens as a transfer with each citizen receiving an equal amount. The continuation value function $v_\theta(B')$ takes into account movements in the price level next period $P'$ caused by the fiscal authority’s choice of nominal bonds $B'$.

The fiscal authority’s first order conditions of this problem are

$$\frac{1 - \tau}{1 - \tau (1 + \epsilon)} = \frac{nA}{g}$$

$$\frac{1 - \tau}{1 - \tau (1 + \epsilon)} = \frac{-n\beta}{q(B')} [\pi v'_H (B') + (1 - \pi) v'_L (B')]$$

The expression $\frac{1 - \tau}{1 - \tau (1 + \epsilon)}$ is the marginal distortionary cost of taxation. The cost is always greater than amount of revenue raised. The first equation equates the marginal cost of raising an additional unit of revenue via taxation with the marginal benefit of spending that revenue on public goods. The second equation equates the marginal cost of raising an additional unit of revenue via taxation with the expected marginal cost of raising the revenue by issuing bonds at the price $q$ (and thus smoothing taxation by pushing the cost into the future).

A benevolent monetary authority cares about the welfare of all citizens while a self-interested monetary authority only cares about its $m$ coalition. In either case, the monetary authority does not have an instrument like the fiscal authority’s transfers that can target individual citizens. The value function for both types of monetary authorities will appear the same, though the representative citizen is either representative of all citizens or of the self-interested
monetary authority’s coalition. Without the ability to use an instrument to target the coalition, the optimization is identical.

The monetary authority chooses $P$ to maximize the welfare of its coalition

$$\max_P \left[ \max_{\tau, g, B'} W_\theta(\tau, g) + \frac{\sum_i T_i}{n} + \beta [\pi v_H (B') + (1 - \pi)v_L (B')] \right]$$

s.t. $T_i \geq 0 \ \forall i, S_\theta(\tau, g, B'; \frac{B}{P}) \geq \sum_i T_i$ \hspace{1cm} (21)

Combining the monetary and fiscal authority’s problems and simplifying we can write this recursively for a given bond level $B$ as

$$v_\theta(B) = \max_P \left[ \max_{\tau, g, B'} W_\theta(\tau, g) + \frac{S_\theta(\tau, g, B'; \frac{B}{P})}{n} + \beta [\pi v_H (B') + (1 - \pi)v_L (B')] \right]$$

s.t. $S_\theta(\tau, g, B'; \frac{B}{P}) \geq 0$ \hspace{1cm} (22)

Solving the monetary authority’s problem for optimal $P$ yields the monetary authority’s pricing function $P(B)$

$$P(B) = \begin{cases} \infty, & \text{if } B > 0 \\ 1, & \text{if } B = 0 \\ \min\{1, \frac{B}{P}\}, & \text{if } B < 0 \end{cases}$$ \hspace{1cm} (23)

This pricing function displays the usual logic of time inconsistency. A self-interested monetary authority behaves identically to a benevolent monetary authority. Members of the monetary authority’s coalition, as well as non-members, desire eliminating nominal debt to lower distortionary taxes thus gain higher welfare.

At the beginning of a period, the monetary authority faces three possibilities for the amount of nominal bonds: a positive amount of nominal bonds (the government is in debt), no nominal bonds, or a negative amount of nominal bonds (the government is owed revenue). If there are a positive amount of nominal bonds, the monetary authority erases the entire real value of the
bonds by setting the price level to infinity. This allows the benevolent fiscal authority to lower taxes since it no longer needs to raise revenue to pay off bonds. Because taxes are distortionary, lower taxes means higher welfare.

If there are no nominal bonds, the price level does not affect welfare. Bonds are the only nominal quantity in the maximization problem. The monetary authority sets the price level to its default value of 1 as explained in Section 3.4. If there are a negative amount of bonds, the monetary authority will deflate until the real value of those bonds equals the Samuelson level $B$. Consumers will demand full compensation for the entire amount of $B$ negative bonds immediately. In that single period, distortionary taxes would have to be extremely high to fund the bond purchases.

Claim 1 The solution for a benevolent fiscal authority with a benevolent or self-interested monetary authority is to issue 0 bonds to raise 0 revenue in every period or to purchase $B$ bonds in the first period and 0 bonds thereafter.

3.3 Structures with a Self-Interested Fiscal Authority

The problem of a self-interested fiscal authority and benevolent monetary authority, as seen in Miller (2016), is

$$v_\theta(B) = \max_P \left[ \max_{\tau, g, B', (T_i)} W_\theta(\tau, g) + \frac{\sum T_i}{m} + \beta \left[ \pi v_H(B') + (1 - \pi) v_L(B') \right] \right]$$  

s.t. $T_i \geq 0 \forall i, S_\theta(\tau, g, B'; \frac{B}{P}) \geq \sum_i T_i$  

(24)

while the problem of a self-interested fiscal authority and self-interested monetary authority is

$$v_\theta(B) = \max_{P, \tau, g, B', (T_i)} \left[ W_\theta(\tau, g) + \frac{\sum T_i}{m} + \beta \left[ \pi v_H(B') + (1 - \pi) v_L(B') \right] \right]$$  

s.t. $T_i \geq 0 \forall i, S_\theta(\tau, g, B'; \frac{B}{P}) \geq \sum_i T_i$  

(25)
The optimization problem differs from the problems of a benevolent fiscal authority in the potential for transfers. I first describe the optimal choices for fiscal variables \( \{\tau_0(B), g_0(B/P), B_0(B/P)\} \) if transfers are zero and if transfers are positive. Then I explain the cutoff level of real bonds \( C_0 \) that determines whether those transfers will be zero or positive. Finally, I derive the optimal choice of the price level \( P \) by the two types of monetary authority. I show that a benevolent monetary authority’s choice depends on the cutoff \( C_0 \) while a self-interested monetary authority’s choice doesn’t depend on \( C_0 \), though the derivative is kinked there.

If transfers are zero, the problem of a self-interested fiscal authority is identical to the problem of a benevolent fiscal authority, hence the optimal choices are given by Equation (20). If transfers are positive, the optimal choices are

\[
\begin{align*}
\frac{n}{m} &= \frac{1 - \tau^*}{1 - \tau^*(1 + \epsilon)} \\
\frac{n}{m} &= \frac{nA}{g^*} \\
B^* &= \arg\max_{B'} \left[ \frac{qB'}{m} + \beta \left[ \pi v_H (B') + (1 - \pi) v_L (B') \right] \right] 
\end{align*}
\]

The left hand side \( \frac{n}{m} \) term represents the amount each individual in the coalition that controls both monetary and fiscal policy will receive as a transfer if an additional unit of revenue is raised. The first equation shows the marginal benefit to coalition members from additional revenue is equal to the marginal cost of raising that additional unit by distortionary taxation. The second equation displays the choice of the government to spend revenue: the marginal benefit of transfers to the governing coalition is equal to the marginal benefit from using that revenue on public good spending. The third equation balances the optimal amount of bond revenue used to fund transfers versus the cost of more bonds in the next period.

When transfers are positive the tax rate, government spending, and level of bonds \( \{\tau^*, g^*, B^*\} \) will be constant. The government will raise revenue from taxes \( \tau^* \) and bonds \( B^* \). It will spend \( g^* \) on the public good. Whatever
revenue is left after that spending will be used to fund transfers. Thus there is a cutoff level of bonds

\[ C_\theta = \text{Rev}_\theta(\tau^*) + qB^* - g^* \]  

(27)

which is the amount of bonds such that \( S_\theta(\tau^*, g^*, B^*; C_\theta) = 0 \). If the level of bonds \( \frac{B}{P} \) after the monetary authority’s choice of \( P \) is above \( C_\theta \) there will be no revenue for transfers. In this region, the optimization problem will be identical to that of the benevolent fiscal authority. If the level of bonds is below \( C_\theta \) there will be transfers while taxes, public good spending and bond issuance are \( \{\tau^*, g^*, B^*\} \) respectively.

The benevolent monetary authority’s choice for the price level is

\[
P_\theta(B) = \begin{cases} 
\frac{B}{C_\theta}, & \text{if } B > C_\theta \\
1, & \text{if } B \leq C_\theta
\end{cases}
\]  

(28)

If the amount of nominal bonds is less than the bond cutoff, there are transfers. If the monetary authority were to increase the price level, the amount of debt the self-interested fiscal authority must repay will go down. Revenue the self-interested fiscal authority had directed to bond repayment will instead go to transfers while taxes and government spending remain constant at \( \tau^* \) and \( g^* \). The total decrease in the real value of nominal bond holdings equals the total increase in transfers. Because utility is quasilinear, overall welfare is unchanged. If the amount of nominal bonds is greater than or equal to the bond cutoff there are no transfers. Increasing the price level results in lower taxes and higher welfare.

Claim 2 The solution for a self-interested fiscal authority with a benevolent monetary authority is to issue \( C_h \) bonds that raise \( \beta(\pi C_h + (1 - \pi)C_l) \) revenue in every period.

The self-interested fiscal authority issues \( C_h \) bonds because that level maximizes the amount of revenue raised while holding taxes and government spending at their constant, starred values. The fiscal authority uses the revenue freed
by inflating away bonds to perfectly smooth taxes across periods. For any realization of the wage shock and any bond level, taxes will be constant at $\tau^*$ and government spending constant at $g^*$. These are the lowest possible taxes and highest possible spending. Thus this equilibrium has the highest possible welfare within the constraints of the model.

The self-interested monetary authority’s choice for the price level is

$$P(B) = \begin{cases} \infty, & \text{if } B > 0 \\ 1, & \text{if } B = 0 \\ 0, & \text{if } B < 0 \end{cases}$$

(29)

A self-interested fiscal authority with a self-interested monetary authority will be unable to raise any revenue from nominal bonds. The self-interested monetary authority will set the price level to infinity for any positive level of bonds. For a real value of bonds above the cutoff $C_\theta$, the logic is the same as before: increasing the price level will result in decreased taxes and therefore increased welfare. New logic applies when the real value of bonds is below the cutoff $C_\theta$: the coalition controlling the self-interested monetary authority will derive a welfare benefit from increasing the price level until the entire real value of the nominal bonds is eliminated even though taxes will remain constant.

The benefit exists because the self-interested monetary authority only cares about the welfare of its coalition. Increasing the price level so the real value of bonds declines below $C_\theta$ decreases the real value of bonds and thus the wealth of all citizens, while taxes and government spending are constant at $\{\tau^*, g^*\}$. However, transfers to the coalition will increase, resulting in a net gain in utility for members of the coalition. For each unit of wealth the coalition loses by raising the price level, the coalition will gain $\frac{\alpha}{m}$ units of transfers. Thus a self-interested monetary authority will eliminate the real value of any positive amount of nominal bonds.

The same logic of increasing transfers to the coalition prevails if there is a negative amount of bonds. The negative value means all citizens owe revenue
to the fiscal authority. The self-interested fiscal authority will distribute that revenue to its coalition while taxes and government spending will be set at the constants \( \{\tau^*, g^*\} \). Setting the price level to 0 maximizes the amount of revenue transferred from the citizenry to the fiscal authority and then back to the coalition. Citizens would expect this extreme deflation and demand infinite compensation for bonds sold to the government.

**Claim 3** *The solution for a self-interested fiscal authority with a self-interested monetary authority is to issue 0 bonds to raise 0 revenue in every period.*

We’ve assumed the self-interested monetary authority shares the same coalition as the self-interested fiscal authority, and thus the same goal of maximizing the welfare of the same subset of citizens. To guarantee Claim 3, there must be sufficient overlap in controlling coalitions to generate the new logic of time inconsistency of the monetary authority. The reason is that inflation eliminates the wealth of the coalition that controls the monetary authority without guaranteeing that coalition transfers from the fiscal authority. Members of the monetary authority’s coalition who aren’t in the fiscal authority’s coalition will vote against decreasing their wealth to increase transfers they won’t receive.

Due to timing, citizens who are members of both the monetary and fiscal coalitions cannot commit to fiscal transfers in order to persuade the monetary coalition. Fiscal decision-making happens after monetary decision-making and no commitment device is available. Thus the results depend on at least the majority of the monetary authority’s coalition (assuming majority rule) also being in the fiscal authority’s coalition. If the overlap is smaller, the self-interested monetary authority will be goal independent from the self-interested fiscal authority and equivalent to a benevolent monetary authority.

### 3.4 Equilibrium Existence

I define and provide an equilibrium for the structures in this paper.
**Definition 1** An equilibrium is a collection of value functions \( v_\theta(B) \), fiscal choices \( \tau_\theta(B) \), monetary choice \( P_\theta(B) \), expectations about the interest rate \( q_\theta(B) \) for \( \theta \in \{ h, l \} \), such that given \( v_\theta(B) \) and \( q_\theta(B) \), the fiscal choices solve the fiscal authority’s problem and given \( v_\theta(B) \) and \( q_\theta(B) \), the monetary choice \( P_\theta(B) \) solves the monetary authority’s problem where given fiscal choices, monetary choice, and \( q_\theta(B) \), \( v_\theta(B) \) are optimal.

We only need the value functions and debt state variable to establish the equilibrium. Everything can be derived, as above, from those.

**Claim 4** With a benevolent fiscal authority, if the value functions \( v_H(B') \), \( v_L(B') \) satisfy Equation 21 and optimal debt satisfies Equation 20, while with a self-interested fiscal authority, if the value functions \( v_H(B') \), \( v_L(B') \) satisfy Equation 23, and optimal debt satisfies Equation 26, then there is an equilibrium in which \( v_H(B') \), \( v_L(B') \) are proposed and accepted in the first round of voting, and fiscal choices \( \tau_\theta(B) \), \( g_\theta(B) \), \( B'_\theta(B) \) and monetary choice \( P_\theta(B) \) are optimal. Existence is then established by showing the joint optimality of the value functions and fiscal and monetary choices.

Due to linear utility there may be a non-singleton set of price levels \( P \) that result in identical welfare. I impose two price selection criteria. First, absent welfare gains, the monetary authority will set the price level to 1. This default price level is a normalization brought on by specifying that bonds return 1 unit of nominal income.

Second, the monetary authority will deviate from \( P = 1 \) only for positive welfare gains. When the monetary authority determines the price level, it will minimize \(|P - 1|\) while maximizing welfare. For a welfare level \( k \) the set \( \{ P \text{ s.t. } v(B) = k \} \) where \( v(B) \) is the welfare function may not be a singleton. As an equilibrium choice, I assume the government always chooses the element of this set that minimizes \(|P - 1|\). These requirements mimic an aversion to inflation and deflation.

Similarly, there may be a non-singleton set of bond amounts \( B' \) that result in the same bond revenue \( qB' = E \left[ \frac{1}{P(B')} \right] B' \). For a revenue level \( k \) the set
3.5 Main Result

Claim 1 shows that both structures with a benevolent fiscal authority cannot support nominal bonds. Claim 2 shows a self-interested fiscal authority with a benevolent monetary authority can support nominal bonds, while Claim 3 shows a self-interested fiscal authority with a self-interested monetary authority cannot. Figure 3 summarizes the four possible structures for monetary and fiscal policy.

**Proposition 1** The only structure that supports nominal bonds has a self-interested fiscal authority and a benevolent monetary authority. This structure leads to the highest welfare among all possible structure.\(^2\)

\(^2\)The welfare result requires that tax smoothing is necessary: an economy with no shocks requires no tax smoothing hence bonds are unnecessary. Specifically, the productivity shocks
The only structure that allows the government to raise revenue from bonds – and has the highest welfare due to the use of the bond revenue for tax smoothing – is the pairing of a self-interested fiscal authority and a benevolent monetary authority. This structure is described in more detail in Claim 2. A benevolent monetary authority sees no welfare gain to eliminating all debt because the self-interested fiscal authority will use its newfound spending freedom to increase transfers to its coalition rather than lower taxes. There is no benefit to inflating away the real value of debt below the cutoff $C_\theta$. The positive level of bonds allows the fiscal authority to smooth shocks.

4 Conclusion

Monetary independence is thought of as a way to overcome inflationary bias. Neither goal nor instrument independence is sufficient to achieve this task. The structure of monetary and fiscal authorities matters. Both the pairing of a self-interested monetary authority with a benevolent fiscal authority and the pairing of a benevolent monetary authority with a self-interested fiscal authority feature goal and instrument independence. Only the latter allows the possibility of nominal bonds.

Goal independence is not enough to lessen inflationary bias because actions that benefit everyone also benefit the coalition of the elected authority. Instrument independence is not enough because, even with explicit instruments and timing, authorities with the same goal coordinate instruments jointly. The combination of goal and instrument independence may not be enough because instruments are not properly constrained.

Goal and instrument independence are necessary to avoid time inconsistency. Advanced economies elect the fiscal authority and attempt to shield the monetary authority from its influence. The reason to use this structure

\[ w_h, w_l \] and corresponding probabilities must be large enough such that

\[
\sum_t \beta^t W_\theta(\tau^*, g^*) > \sum_t \beta^t W_\theta(\tau_0, g_0)
\]  

(30)

where $\tau_0, g_0$ are the tax rate and public good spending that prevail without bonds.
is its unique ability to alleviate time inconsistency and allow bonds for tax smoothing. The uniqueness of this structure explains why monetary independence is so important. Setting up a structure with an elected fiscal authority and allowing it to capture of the monetary authority undoes any benefits the monetary authority provides.
References


Appendix

A.1 Proof of Claim

To do this it suffices to show that

\[
P(B) = \begin{cases} 
\infty, & \text{if } B > 0 \\
1, & \text{if } B = 0 \\
\frac{B}{B}, & \text{if } B < B < 0 \\
1, & \text{if } B \leq B
\end{cases}
\]  

(31)

Concentrating on the case of \( B > 0 \), I show that welfare always increases as the real value of bonds is diminished, thus there is always an incentive for the monetary authority to increase the price level.

To find the appropriate price level, choose \( B_0 > 0 \). I will build a non-optimal function \( \phi_\theta(B) \) that equals \( v_\theta(B) \) at \( B_0 \) but is less elsewhere (and strictly concave). This will fulfill the conditions of Theorem 4.10 of Stokey et al. (1989) stating that derivatives of \( v_\theta(B) \) are equal to the derivatives of \( \phi_\theta(B) \) at \( B_0 \). For clarity and notational simplicity let \( b = \frac{B}{P(B)} \) and \( b_0 = \frac{B_0}{P(B_0)} \).

Choose \( B \) from a neighborhood of \( B_0 \). Define

\[
g(b) = \text{Rev}(\tau(b_0)) + qB'(b_0) - b
\]

(32)

which is a non-optimal amount of government spending while still fulfilling debt repayment obligations. The amount of transfers will be the residual after paying back \( b \) bonds

\[
S_\theta(\tau_\theta(b_0), g(b), B'(b_0); b) = \text{Rev}(\tau(b_0)) + qB'(b_0) - g(b) - b
\]

(33)
Define the non-optimal utility function to be

\[
\phi_\theta(B) = \max_P W(\tau(b_0), g(b)) + S_\theta(\tau_\theta(b_0), g(b), B'(b_0); b) \\
+ \beta [\pi v_H (B'(b_0)) + (1 - \pi) v_L (B'(b_0))] \\
= \max_P \Gamma_\theta(B)
\]  

(34)

Expand the indirect utility and transfers terms. The terms dependent on \(P\) are the direct utility benefit of government spending, the bond holdings of the household in the current period, and transfers. Differentiate the right hand side, noting that the terms dependent on \(P\) in transfers will cancel, and find

\[
\frac{\partial \Gamma_\theta(B)}{\partial P} = -\frac{B}{P^2} + A \left( \frac{B}{P^2} \right) \\
= -\frac{B}{P^2} + \left[ \frac{1 - \tau \left( \frac{B}{P} \right)}{1 - \tau \left( \frac{B}{P} \right)} (1 + \epsilon) \right] \left( \frac{B}{nP^2} \right) \\
= \left[ \frac{\epsilon \tau \left( \frac{B}{P} \right)}{1 - \tau \left( \frac{B}{P} \right)} (1 + \epsilon) \right] \left( \frac{B}{nP^2} \right) > 0
\]  

(35)

where I’ve substituted in the first order condition of the fiscal authority. Taking the second derivative confirms the necessary conditions. A similar construction proves the case for \(B < 0\).

A.2 Proof of Claims 2 and 3

Using the same construction as in the proof of Claim 1 following Miller (2016) the first order condition for the benevolent monetary authority is

\[
\frac{\partial \Gamma_\theta(B)}{\partial P} = \begin{cases} \\
\left[ \frac{\epsilon \tau \left( \frac{B}{P} \right)}{1 - \tau \left( \frac{B}{P} \right)} (1 + \epsilon) \right] \left( \frac{B}{nP^2} \right) & \text{if } B > C_\theta \\
0, & \text{if } B < C_\theta
\end{cases}
\] 

(36)

If \(\frac{B}{P} > C_\theta\) the self-interested fiscal authority’s problem is identical to the benevolent fiscal authority’s problem hence the derivative is equal. When \(\frac{B}{P} < C_\theta\) the derivative can be taken directly from the definition of \(v(B)\).
Increasing the price level causes no change in taxes or government spending which are pegged at $\tau^*, g^*$ respectively while real wealth declines and transfers to the coalition increase by equal amounts. Because utility is linear taking 1 unit of wealth from $n$ citizens while giving $\frac{n}{m}$ to $m$ citizens results in identical welfare from the perspective of the benevolent monetary authority.

Claim 2 says that a self-interested fiscal authority will issue $C_h$ bonds, equivalently, revenue is maximized by issuing $C_h$ bonds. Revenue from issuing bonds is used to either lower the current tax rate or increase transfers. Both of these result in gains for the self-interested fiscal authority. Hence the self-interested fiscal authority will attempt to maximize bond revenue. Issuing more bonds than $C_h$ will result in no additional revenue due to an offsetting rise in the price level, issuing fewer bonds than $C_h$ will result in foregone revenue if tomorrow has high productivity.

The first order condition for the self-interested monetary authority is

$$\frac{\partial \Gamma_{\theta}(B)}{\partial P} = \begin{cases} \frac{\epsilon \tau^2(B)}{1-\tau^2(\frac{B}{P})(1+\epsilon)} \frac{B}{nP^2}, & \text{if } B > C_\theta \\ \left(\frac{n}{m} - 1\right) \frac{B}{nP^2}, & \text{if } B < C_\theta \end{cases}$$

(37)

The case $B < C_\theta$ arises from the equivalence of government debt and transfers in a consumer’s budget constraint: both are income. Receiving a transfer is identical to holding government debt. Increasing the price level decreases the real value of the nominal government bonds every consumer holds. The total decrease in debt will equal the total increase in transfers that benefit the coalition that controls the monetary authority.

A self-interested monetary authority doesn’t average those transfers across the entire population. The monetary authority only cares about the effect on the coalition that controls it. Increasing the price level decreases the amount the government has to repay everyone while increasing the transfers to the coalition controlling the monetary authority. Inflation is in effect a lump sum tax on all citizens to fund transfers to the coalition.
A.2.1 Proof of Claim 4

I follow the outline of Barseghyan et al. (2013) (specifically Propositions 1, 2 and 3) and Miller (2016) (specifically Claim 4) to show the value functions are properly defined and converge in the case where both the monetary and fiscal authority are self-interested. The situation where only the monetary authority is self-interested and chooses only \( P \) is both simpler and similar. The situation where only the fiscal authority is self-interested is shown in Miller (2016) in Claim 4.

Define \( \tilde{v}_\theta \) as the value function and \( \tilde{B}' \) as the bond level such that the value function solves the optimization problem given \( \tilde{B}' \) while the bond level solves the appropriate optimality condition given \( \tilde{v}_\theta \). Define \( \{ \tilde{P}(\frac{B}{P}), \tilde{\tau}(\frac{B}{P}), \tilde{g}(\frac{B}{P}), \tilde{B}'(\frac{B}{P}) \} \) to be the fiscal policy choices conditional on the value function \( \tilde{v}_\theta \) and bond level \( \tilde{B}' \).

Because we’re looking at an equilibrium where the first proposal is identical and accepted in every proposal round I don’t include superscripts to denote the round in which a proposal takes place. Define transfers for each round \( r \in \{1, \ldots, T-1\} \) as
\[
T^r_\theta = \frac{S_\theta(\tau, g, B'; \frac{B}{P})}{n}
\]  
and for round \( T \) where failure to approve a proposal results in the appointment of a dictator and a default proposal
\[
T^T_\theta = v^{T+1}_\theta(B) - W_\theta(\tau, g) - \beta [\pi v_H(B') + (1-\pi)v_L(B')]
\]
where \( v^{T+1}_\theta(B) \) is the default proposal.

By construction the value functions for each proposal round are identical and equal to the proposed \( \tilde{v}_\theta(B) \).

We can now show that the proposals and value functions describe an equilibrium by showing the joint optimality of the value functions and proposals.
In each round, the proposals must solve

\[
v_\theta(B) = \max_{P,\tau,g,B'} \left[ W_\theta(\tau, g) + \left[ S_\theta(\tau, g, B'; \frac{B}{P}) - (m - 1)T \right] + \beta \left[ \pi v_H(B') + (1 - \pi)v_L(B') \right] \right]
\]

subject to

\[
W_\theta(\tau, g) + T + \beta \left[ \pi v_H(B') + (1 - \pi)v_L(B') \right] \geq \Gamma_r^{T+1}(b)
\]

\[
S_\theta(\tau, g, B'; \frac{B}{P}) \geq (m - 1)T, T \geq 0
\]

where \(\Gamma_r = \tilde{v}_\theta(b)\) for \(r \in \{1, \ldots, T - 1\}\), \(\Gamma^{T+1} = v_\theta^{T+1}(B)\) is the set of possible continuation values if a proposal is not approved. The first constraint on the proposal is the incentive compatibility constraint for citizens. It ensures citizens vote for a proposal if it makes them at least as well off as they expect to be if they wait for the next round and next proposal.

For a given proposal round suppose \((\hat{P}, \hat{\tau}, \hat{g}, \hat{B}', \hat{T})\) is the proposal. This means the proposal solves

\[
\max_{P,\tau,g,B',T} \left[ W_\theta(\tau, g) + \left[ S_\theta(\tau, g, B'; \frac{B}{P}) - (m - 1)T \right] + \beta \left[ \pi v_H(B') + (1 - \pi)v_L(B') \right] \right]
\]

subject to

\[
S_\theta(\tau, g, B'; \frac{B}{P}) \geq (m - 1)T, T \geq 0
\]

which defines the remaining surplus as

\[
\hat{T} = \tilde{v}_\theta(B) - W_\theta(\hat{\tau}, \hat{g}) - \beta \left[ \pi \tilde{v}_H(\hat{B}) + (1 - \pi)\tilde{v}_L(\hat{B}) \right]
\]

Suppose there is a proposal \((P^o, \tau^o, g^o, B'^o, T^o)\) that results in a higher value for the proposer. I will construct a contradiction using this new set of proposals and the definition of \(T^o\). Since the new proposals result in higher values, we know that

\[
T^o \geq \tilde{v}_\theta(B) - W_\theta(\tau^o, g^o) - \beta \left[ \pi \tilde{v}_H(B'^o) + (1 - \pi)\tilde{v}_L(B'^o) \right]
\]
\[
W_\theta(\tau^0, g^0) + S_\theta(\tau^0, g^0, B'^0; \frac{B}{P}) - (m - 1)T^0 + \beta \left[ \pi \tilde{v}_H(B'^0) + (1 - \pi)\tilde{v}_L(B'^0) \right] \\
\leq q \left( W_\theta(\tau^0, g^0) + \beta \left[ \pi \tilde{v}_H(B'^0) + (1 - \pi)\tilde{v}_L(B'^0) \right] \right) + S_\theta(\tau^0, g^0, B'^0; \frac{B}{P}).
\]

(44)

But the last equation (without the multiplication by \( q \)) is the objective function (\( \hat{P}, \hat{\tau}, \hat{g}, \hat{B}' \)) are defined as solving. To show the equilibrium exists we need to show that \( v_\theta \) and the proposals are jointly optimal. This means that \( v_\theta \) is the solution to

\[
v_\theta(B) = \max_{P, \tau, g, B'} \left[ \frac{W_\theta(\tau, g) + S_\theta(\tau, g, B'; \frac{B}{P})}{m} + \beta \left[ \pi v_H(B') + (1 - \pi)v_L(B') \right] \right]
\mathrm{s.t.}\ T_i \geq 0 \ \forall i, S_\theta(\tau, g, B'; \frac{B}{P}) \geq \sum_i T_i
\]

(45)

given the proposals \((P^*, \tau^*, g^*, B'^*)\) and that given \( v_\theta \) the proposals solve the requisite optimality conditions.

Let \( F \) be the set of real valued, continuous, concave functions \( v \) over the domain of possible bond values \([B, B]\).

For \( z_0 = [\mathrm{Rev}_L(\tau^*) - g^*, B] \) define \( N^\theta_{z_0} \) from \( F \times F \to F \) as

\[
N^\theta_{z_0}(v_h, v_l)(B) = \max_{P, \tau, g, B'} \left[ \frac{W_\theta(\tau, g) + S_\theta(\tau, g, B'; \frac{B}{P})}{m} + \beta \left[ \pi v_H(B') + (1 - \pi)v_L(B') \right] \right]
\mathrm{s.t.}\ T_i \geq 0 \ \forall i, S_\theta(\tau, g, B'; \frac{B}{P}) \geq \sum_i T_i
\]

(46)

For conciseness let \( z = (z_H, z_L) \) be the vector of possible bond levels for either realization of the productivity shock. Define \( N_z(v_h, v_l)(B) = (N^H_{z_H}(v_h, v_l)(B), N^{L}_{z_L}(v_h, v_l)(B)) \) from \( F \times F \to F \times F \) as the optimal choices for either realization of the productivity shock. For any \( z \), \( N_z \) is a contraction with a unique fixed point \( v_z \).
We can use this fixed point to define

\[
M_\theta(z) = \arg \max_{B'} \left[ \frac{qB'}{m} + \beta \left[ \pi v_{Hz}(B') + (1 - \pi)v_{Lz}(B') \right] \right]
\]  

(47)

where the fixed point is used for computing the possible continuation values.

Let \( M(z) = M_H(z) \times M_L(z) \). An equilibrium is a fixed point of \( M(z) \) that takes a bond level \( z \) to itself given the fixed point \( v_z \). Applying Kakutani’s Fixed Point Theorem proves the result.

Differentiability of the value function comes from a similar construction as the proof of Claim 1. Using our fixed point \( v_z \) we can choose a non-optimal bond level \( B_0 \) and show differentiability. In the region with transfers, this is direct, in the region without transfers the construction proceeds identically to Claim 1.