Maybe “honor thy father and thy mother”: uncertain family aid and the design of social long term care insurance

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July 2016, revised December 2016

*Financial support from the Chaire “Marché des risques et creation de valeur” of the FdR/SCOR is gratefully acknowledged. We thank Motohiro Sato whose suggestions inspired the representation of uncertain altruism we use.
Abstract

We study the role and design of long-term care insurance programs when informal care is uncertain. Insurance crowds out informal care, both at the intensive and the extensive margins. We consider three types of public insurance policies: (i) a topping up (TU) scheme which can be supplemented, (ii) an opting out (OO) scheme which is exclusive, and (iii) a mixed policy. OO crowds out informal care solely at the extensive margin, whereas TU involves crowding out also at the intensive margin. However, OO may exacerbate crowding out at the extensive margin. An appropriately designed mixed policy can mitigate these distortions.

**JEL classification:** H2, H5.

**Keywords:** Long term care, uncertain altruism, private insurance, public insurance, topping up, opting out.
1 Introduction

Long-term care (LTC) concerns people who depend on help for daily activities. It is different from (albeit often complementary to) health care, and particularly terminal care or hospice care. Dependent individuals do not just need medical care, but also need help in their everyday life. Providing this assistance is very labor intensive and may be very costly, especially when severe dependence calls for institutional care.

Dependence represents a significant financial risk of which only a small part is typically covered by social insurance. Private insurance markets are also thin. While medical acts are typically reimbursed, at least in part, by health insurance, LTC is usually not covered. As a consequence, individuals rely on their savings or on informal care provided by family members, which is estimated to represent about two thirds of total LTC.\(^1\) This is inefficient and often insufficient, since it leaves individuals who for whatever reason cannot count on family solidarity without proper care.

The importance of informal care is likely to decrease during the decades to come because of various societal trends. These include population ageing, drastic changes in family values and increased female labor force participation. Consequently, the need of private and social LTC insurance will become more pressing.

Irrespective of these long-run trends, informal care is in any event subject to many random shocks. There are pure demographic factors such as widowhood, the absence or the loss of children. Divorce and migration can also be put in this category. Other shocks are conflicts within the family or financial problems incurred by children, which prevent them from helping their parents. While private insurance markets could potentially provide coverage against the risk of dependence \textit{per se}, the uncertainty associated with the level of informal care appears to be a mostly uninsurable risk. There exists no good private insurance mechanism to protect individuals against all sorts of default of family altruism, in particular because family care is by definition informal and thus largely unobservable and subject to moral hazard. This market failure creates a \textit{potential} role for public intervention. However, it is not likely that public administrations have better information than parents about their prospects to receive informal care. Consequently, public intervention...

\(^1\)See Norton (2000).
won’t lead to a first-best outcome and the design of second-best policies is not trivial.

This paper studies the role and design of LTC policy when informal care is uncertain. This uncertainty is represented by a single parameter, $\beta$, referred to as the child’s degree of altruism. One can also think about this as the (inverse of the) cost of providing care. This parameter is not known to parents when they make their savings and insurance decisions. We concentrate on a single generation of parents over their life-cycle. When young, they work, consume, and save for their retirement. When old, they face the risk of becoming dependent. Furthermore, they don’t know the level of informal care, if any, they can expect from their children in case of dependence. This is determined by the parameter $\beta$ which is randomly distributed over some interval.

We consider two types of LTC policies. The first one, referred to as “topping up” ($TU$) provides a transfer to dependent elderly, which is non-exclusive and can be supplemented by informal care and by market care financed by savings. The second one, is an “opting out” scheme ($OO$); it provides care which is exclusive and cannot be topped up. One can think of the $TU$ policy as a cash transfer or as services provided at home and that can be supplemented by market and informal care. The $OO$ scheme can for instance provide free or subsidized institutional care. Finally, we study a mixed policy which combines both approaches, and lets parents choose between, say, a monetary help for care provided at home (a $TU$ scheme) and nursing home care provided on an $OO$ basis. In practice, different LTC schemes may coexist within a given country.\(^2\) In some countries, such as Austria, Finland, and Italy, LTC policy relies heavily on cash benefits. Others countries, such as the Scandinavian ones, use mostly in-kind transfers, consisting care provided at home or in institutions. Institutionalized individuals may have to pay a rent, and may be granted a personal need allowance to pay for residual consumption. Even in Scandinavia, where LTC insurance is based on the provision of formal care, dependent individuals continue to rely heavily on informal care.\(^3\) Finally, in some countries, such as Germany, individuals can choose between in-kind or in-cash transfers.

A major concern raised by LTC policies is that of crowding out of informal care.\(^4\) This

\(^2\)For an overview of different policies and financing models in the EU, see Lipszyc et al. (2012) and European Commission (2013).
\(^3\)See Karlsson et al. (2010).
may make public LTC insurance ineffective for some persons and overall more expensive. Within the context of informal care crowding out may occur both at the intensive and the extensive margins. Intensive margin refers to the reduction of informal care, possibly on a one by one basis, for parents who continue to receive aid from their children, even when social LTC is available. Crowding out at the extensive margin, on the other hand, occurs when some children are dissuaded from providing any informal care. The two types of policies we consider have different impacts on informal care. $TU$ will involve crowding out both at the intensive and the extensive margins, whereas $OO$ will crowd out informal care solely at the extensive margin.

The distinction between $TU$ and $OO$ has been widely studied in the literature about in-kind vs cash transfers.\(^5\) For instance, it has been shown to be relevant in the context of education and health both from a normative and a positive perspective.\(^6\) In our context, as $OO$ only crowds out informal care at the extensive margin, one might be tempted to think that this makes $OO$ the dominant policy. However, we shall show that this is not necessarily true, as this policy may exacerbate the extensive margin crowding out. Interestingly, when a mixed scheme is used, the policies interact in a nontrivial way. When combined, the crowding out effects induced by each of the policies do not simply add up. Quite the opposite, the policies can effectively be used to neutralize their respective distortions. For instance variations in the policies can be designed so that the marginal level of altruism (above which children provide care) and savings are not affected.

Throughout the paper we concentrate on intra-generational issues; the cost of the LTC program is borne by the generation who also benefits from it. In other words we consider a single generation of parents. The role of children is limited to their decision on the provision of informal care to their parents. The welfare of the grown-up children does not figure in social welfare, which accounts only for the expected lifecycle utility of parents. Note that including caregivers’ utility in social welfare would not affect the fundamental tradeoffs involved in the design of the considered LTC policies.\(^7\) It would simply imply

\(^5\)For a review of the literature, see Currie and Galvari (2008).
\(^6\)On the normative side, for instance, Blomquist and Christiansen (1998) show that both regimes can be optimal (to supplement an optimal income tax) depending on whether the demand for the publicly provided good increases or decreases with labor. From a positive perspective, $TU$ regimes may emerge from majority voting rules, as shown by Epple and Romano (1996).
\(^7\)As long as we maintain the assumption that each generation pays for its own LTC insurance.
that informal care no longer comes for free, which in turn mitigates the adverse effects of crowding out.\textsuperscript{8}

The issue of uncertain altruism has previously been studied by Cremer et al. (2012) and (2014). Both of these papers concentrate on the case where altruism is a binary variable. Children are either altruistic at some known degree or not altruistic at all. The first paper considers both $TU$ and $OO$, while the second one concentrates on $OO$ but accounts for the possibility that the probability that children provide care is endogenous and can be affected by parents’ decisions. None of them studies mixed policies. The current paper considers a continuous distribution of the altruism parameter. This is not just a methodological exercise, but has important practical implications for the results and the tradeoffs that are involved. The very distinction between crowding out at the extensive and intensive margins is not meaningful in the binary model, but turns out to play a fundamental role for policy design when the distribution of the altruism parameter is continuous. This is particularly true in the case of mixed policies; the tradeoffs we have identified there are completely obscured in the binary model.

The paper is organized as follows. In Section 2 we present the model and two interesting benchmarks: the laissez-faire and the full-information allocations. We characterize the optimal $TU$ and $OO$ policies in Sections 3 and 4 respectively. In Section 5 we compare the two policies regimes and provide a sufficient condition for $OO$ to dominate. We discuss the case with private LTC insurance and the mixed regime in Sections 6 and 7.

2 The model

Assume that elderly parents face the risk of becoming dependent. In that case, they may receive informal care from their children depending on their degree of altruism. All parents are identical ex ante and we concentrate on a single generation.

The sequence of events is as follows. In period 0, the government formulates and announces its tax/transfer policy; this is the first stage of our game. The second stage is played in period 1 where young working parents decide on their saving. Finally, in period

\textsuperscript{8}The extent of this would depend on the exact specification of social welfare. The crucial issue from that perspective is how to account for the altruistic term in welfare. Under a strict utilitarian approach it would be fully included which raises the well known problem of “double counting”.

4
2, parents have grown old, are retired, and may be dependent. When parents are healthy the game is over and no further decisions are to be taken. They simply consume their saving. However, when the parents are dependent, we move to stage 3 where the children, who have turned into working adults, decide how much informal care (if any) they want to provide.

Parents face two types of uncertainty. One concerns their health in old age; they may be either “dependent” or “independent”. Denote the probability of dependence by $\pi$ and assume that it is exogenously given. The second source of uncertainty concerns the degree of altruism $\beta \geq 0$ of their children. It is not known to parents in period 1, when they have to make their savings decisions. The random variable $\beta$ is distributed according to $F(\beta)$, with density $f(\beta)$. Children who are not altruistic towards their parents have $\beta = 0$. For simplicity, we assume that $F$ is concave, which implies that $F(\beta) > \beta f(\beta)$.

Parents have preferences over consumption when young, $c \geq 0$, consumption when old and healthy, $d \geq 0$, and consumption, including LTC services, when old and dependent, $m \geq 0$. There is no disutility associated with working. Parents’ preferences are quasilinear in consumption when young. Risk aversion is introduced through the concavity of second period state dependent utilities.

The policy consists in the provision of dependence assistance, $g$, financed by a linear (proportional) tax at rate of $\tau$ on the parents’ wage, $w$. We shall refer to $g$ as the LTC insurance benefit. We rule out private insurance in the basic model, but we will introduce it in Section 6. Denote the level of informal care by $a$, savings by $s$, and set rate interest rate on savings equal to zero.

Before turning to policy design, we study the *laissez-faire*, which is an interesting benchmark. We proceed by backward induction and start by studying the last stage of the decision making process. This is when the grown-up children decide on the extent of their help to their parents, if any.

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*9*We rule out $\beta < 0$, which represents a case where children are happier if their parents are worse off.

*10*This condition is sufficient (but not always necessary) for most of the SOC to be satisfied and for some comparative statics results. We shall point out explicitly where and how it used.

*11*In the *laissez-faire* there is no first stage. To be consistent with the next section we refer to the relevant stages as 2 and 3.
2.1 Laissez-faire

In this section we assume that the government does not provide any form of LTC insurance. The parent’s expected utility is given by

\[ EU = wT - s + (1 - \pi)U(s) + \pi E[H(m)], \]  \hspace{1cm} (1)

with \( m = s + a^*(\beta, s) \), where \( s \) is saving, while \( a^*(\beta, s) \geq 0 \) is informal care provided by children, which depends on their degree of altruism \( \beta \). Assume that \( H' > 0, H'' < 0, H(0) = 0, H'(0) = \infty \), and that the same properties hold for \( U \). Further, for any given \( x \) we have \( U'(x) < H'(x) \). To understand this condition consider the simplest case where dependence implies a monetary loss \( L \) so that \( H(x) = U(x - L) \).

We shall assume that the grown-up children too have quasilinear preferences represented by

\[ u = \begin{cases} y - a + \beta H(m) & \text{if the parent is dependent,} \\ y - a & \text{if the parent is not independent,} \end{cases} \]  \hspace{1cm} (2)

where \( y \) denotes their income. Altruism is relevant only when the child’s parent is effectively dependent. Healthy elderly parents consume their savings; they neither give nor receive any transfer.

2.1.1 Stage 3: The child’s choice

The altruistic children allocate an amount \( a \) of their income \( y \) to assist their dependent parents, given the parents’ savings \( s \). Its optimal level, \( a^* \), is found through the maximization of equation (2). The first-order condition with respect to \( a \) is, assuming an interior solution,

\[ -1 + \beta H'(s + a) = 0. \]

Define \( \beta_0(s) \) such that

\[ 1 = \beta_0 H'(s). \]  \hspace{1cm} (3)

This function represents the minimum level of \( \beta \) for which a positive level of care is provided. Not surprisingly, we have

\[ \frac{\partial \beta_0}{\partial s} = -\frac{\beta_0 H''}{H'} > 0, \]
implying that an increase in parents’ savings reduces the probability of children providing care in case of dependence. It follows from condition (3) that when \( \beta \geq \beta_0 > 0 \), \( a^* \) satisfies

\[
m = s + a^* = (H')^{-1} \left( \frac{1}{\beta} \right).
\]  

Conversely, when \( \beta < \beta_0 \), \( a^* = 0 \) and \( m = s \). In the laissez-faire, the consumption of dependent parents is then equal to

\[
m(\beta) = \begin{cases} 
(H')^{-1} \left( \frac{1}{\beta} \right) & \text{if } \beta \geq \beta_0, \\
m_0 = s & \text{if } \beta < \beta_0.
\end{cases}
\]  

This expression shows that savings crowds out informal care in two ways. First, by increasing \( \beta_0 \), it makes it less likely that a positive level of care is provided (crowding out at the extensive margin). Second, when informal care is provided it is crowded out on a one to one basis by savings, as long as the solution remains interior (crowding out at the intensive margin). Differentiating (4) yields

\[
\frac{\partial m}{\partial \beta} = \begin{cases} 
\frac{-1}{\beta^2 H''(m)} & \text{if } \beta \geq \beta_0, \\
0 & \text{if } \beta < \beta_0,
\end{cases}
\]  

where the first line is positive from the concavity of \( H \). As expected, a parent’s total consumption when dependent increases with the degree of altruism of the child. Figure 1, which represents equation (5), illustrates the consumption level of dependent parents as a function of children’s altruism.

2.1.2 Stage 2: The parent’s choice

Recall that the parent may experience two states of nature when retired: dependence with probability \( \pi \) and autonomy with probability \( 1 - \pi \). Substituting for \( a^* \) from (4) in the parent’s expected utility function (1), we have

\[
EU = w \bar{T} - s + (1 - \pi) U(s) + \pi \left[ \int_0^{\beta_0} H(s) dF(\beta) + \int_{\beta_0}^{\infty} H(m(\beta)) dF(\beta) \right],
\]

\[
= w \bar{T} - s + (1 - \pi) U(s) + \pi \left[ H(s) F(\beta_0) + \int_{\beta_0}^{\infty} H(m(\beta)) dF(\beta) \right].
\]

The function \( m(\beta) \) is not differentiable at \( \beta = \beta_0 \). To avoid cumbersome notation we use \( \partial m/\partial \beta \) for the right derivative at this point.
Maximizing $EU$ with respect to $s$, and assuming an interior solution the optimal value of $s$ satisfies\(^\text{13}\)

\[
(1 - \pi) U'(s) + \pi F(\beta_0) H'(s) = 1.
\]

Note that there are no terms involving $\partial m/\partial s$. The derivatives with respect to $\beta_0$ cancel out because $m(\beta_0) = s$. Expression (6) simply states that the expected benefits of saving must be equal to its cost, which is equal to 1. Saving provides benefits when the parent is healthy; this is represented by the first term on the RHS. The second term corresponds to the benefits enjoyed by the dependent elderly who do not receive informal care. When the dependent parents receive informal care, saving has no benefit because of the crowding out.

Observe that, since $H'(s) > U'(s)$, as long as $F(\beta_0) < 1$ equation (6) implies that $H'(m_0) > 1$ so that as expected the *laissez-faire* leaves dependent individuals who do not receive formal care underinsured.

The SOC is given by

\[
(1 - \pi) U''(s) + \pi F(\beta_0) H''(s) + \pi f(\beta_0) H'(s) \frac{\partial \beta_0}{\partial s} < 0,
\]

or, substituting for $\partial \beta_0 / \partial s$

\[
(1 - \pi) U''(s) + \pi H''(s) [F(\beta_0) - \beta_0 f(\beta_0)] < 0,
\]

for which the concavity of $F(\beta)$ represents a *sufficient* condition.

\(^{13}\)A corner solution at $s = 0$ can be excluded by the assumption that $U'(0) = \infty$. However, a corner solution at $s = wT$, yielding $c = 0$ cannot be ruled out. To avoid a tedious and not very insightful multiplication of cases we assume throughout the paper that the constraint $c \geq 0$ is not binding in equilibrium (even when first period income is taxed to finance social LTC).
2.2 Full-information solution

To assess this equilibrium and determine the need for policy intervention let us briefly examine the full-information allocation. We define this as the allocation that maximizes the expected utility of the parent *taking the aid behavior as given*, but assuming that \( \beta \) is observable. In other words, it is possible to insure individuals both against dependence and against the failure of altruism, because the payment to dependent parents can be a function of \( \beta \). However, \( a \) is not publicly observable and children cannot be "forced" to provide informal care. In that case, we maximize

\[
EU = wT - (1 - \pi)d - \pi F(\beta_0)m_0 + (1 - \pi)U(d) + \pi \left[ H(m_0)F(\beta_0) + \int_{\beta_0}^{\infty} H(m(\beta))dF(\beta) \right].
\]

with respect to \( d \), \( m_0 \), and \( \beta_0 \). The FOC with respect to \( d \) and \( m_0 \) imply

\[
U'(d^F) = H'(m_0^F) = 1,
\]

and differentiating with respect to \( \beta_0 \) yields

\[
H(m(\beta_0)) = H(m_0^F),
\]

where the superscript \( F \) is used to denote the solution. The latter condition implies that dependent parents rely on informal care when \( m(\beta) > m_0^F \), where \( m_0 = H'^{-1}(1) \) is the solution to equation (7). Using the definition of \( m(\beta) \) in (5), we can conclude that under full information \( \beta_0^F = 1 \). Only children with an altruism parameter greater than one should provide help. According to (7), all other parents should consume \( m_0^F \) in case of dependence. In other words, if the government was able to observe \( \beta \), it would be optimal to provide full insurance to parents whose children put more weight on their own utility \((\beta < 1)\), and let the remaining children help their parents. The latter children would provide more than full insurance because they value the utility of the parents more than their own.

When \( \beta \) is observable, this policy can be easily implemented by a transfer \( g = m_0^F - d^F \) to all dependent parents whose children’s altruism parameter is lower than one, financed by a uniform tax of \( \pi F(\beta_0^F)(m_0^F - d^F) \). Since a child with altruism \( \beta \) provides informal care only if \( \beta H'(m_0^F) > 1 \), all children with \( \beta \leq 1 \) would provide no informal care. Conversely, all children with \( \beta > 0 \) would provide informal care according to \( \beta H'(s + a) = 1 \). Under
this policy, savings are given by the first order condition of the parents $(1 - \pi)U'(s) + \pi H'(g + s) = 1$. Since $H'(g + s) = H'(m_0) = 1$, savings are such that $U'(s) = 1$, and the intertemporal allocation of consumption is the one of the full-information solution.

However, when $\beta$ is not observable this solution cannot be achieved and we have to study policy design in a second-best world. We now study three second-best policies. In the first one, referred to as topping up (TU), the transfer to the dependent parents is conditional on dependence only. It can be supplemented by informal and market care. Under the second one, referred to as opting out (OO), LTC benefits are exclusive and cannot be topped up. Finally, we consider a mixed policy which combines cash benefits that can be topped up with in-kind care that is exclusive. In that case dependent parents can choose their preferred regime.

Observe that by restricting our attention to these policies we implicitly assume that informal care, $a$, is not observable. The only exception is that $a = 0$ can be enforced to implement an OO policy. If $a$ were fully observable, we could of course do better by using a nonlinear transfer scheme $g(a)$ to screen for the $\beta$’s. This would amount to characterizing the optimal incentive compatible mechanism of which TU, OO, and mixed policies are special cases. However, as explained by Norton (2000) family care is by definition informal and thus typically not observable.\(^\text{14}\)

### 3 Topping up

In this section we assume that the government provides LTC insurance. The transfer to the dependent elderly, $g$, is non-exclusive in the sense that it can be topped up by $a$ and $s$. The parent’s expected utility is now given by

\[
EU^{TU} = w (1 - \tau) \bar{T} - s + (1 - \pi) U(s) + \pi E[H(s + g + a^*(\beta, s, g))],
\]  

where $a^*(\beta, s, g) \geq 0$ is care provided by children, which as shown in the next subsection now also depends on $g$. Preferences of grown-up children continue to be represented by equation (2), with $m$ redefined as $m = s + g + a$.

Once again we proceed by backward induction and start with the last stage.

\(^{14}\)This is so much the case that even our statistical knowledge of the extent of informal care is rather imperfect.
3.1 Stage 3: The child’s choice

The altruistic children allocate an amount $a$ of their income $y$ to assist their dependent parents (given the parents’ savings $s$ and the government’s provision of $g$). Its optimal level, $a^*$, is found through the maximization of equation (2). The first-order condition with respect to $a$ is, assuming an interior solution,

$$-1 + \beta H'(s + g + a) = 0.$$ 

Define $\tilde{\beta}(s + g)$ such that

$$1 = \tilde{\beta}H'(s + g).$$ 

Comparing (3) with (9), we obtain that $\tilde{\beta} > \beta_0$ for all $g > 0$. It follows from condition (9) that when $\beta \geq \tilde{\beta}$, $a^*$ satisfies

$$m = s + g + a^* = (H')^{-1}\left(\frac{1}{\beta}\right) = m(\beta).$$ 

As depicted by the solid line in Figure 2, if $\beta \geq \tilde{\beta}$, the consumption of dependent parents $m(\beta)$ is exactly the same as in the laissez-faire. When children’s altruism is in that range informal care is fully crowded out by government assistance. For lower levels of altruism, when $\beta < \tilde{\beta}$, no formal care is provided, $a^* = 0$ and $m = s + g > m(\beta)$. As usual, crowding out stops when caregivers are brought to a corner solution. For these parents, $g$ is effectively increasing the total care they receive and we have $\partial m/\partial g = 1$. Finally, observe that

$$\frac{\partial \tilde{\beta}}{\partial (s + g)} = -\frac{\tilde{\beta}H''}{H'} > 0.$$ 

In words, as the total amount of formal care increases, the degree of altruism necessary to yield a positive level of informal care increases.

3.2 Stage 2: The parent’s choice

Recall that parents are dependent with probability $\pi$ and autonomous with probability $(1 - \pi)$. Substituting for $a^*$ from (10) in the parent’s expected utility function (8), we have

$$EU^{TU} = w(1 - \tau)\overline{T} - s + (1 - \pi)U(s) + \pi\left[\int_{0}^{\tilde{\beta}} H(s + g) dF(\beta) + \int_{\tilde{\beta}}^{\infty} H(m(\beta)) dF(\beta)\right]$$

$$= w(1 - \tau)\overline{T} - s + (1 - \pi)U(s) + \pi\left[H(s + g)F(\tilde{\beta}) + \int_{\tilde{\beta}}^{\infty} H(m(\beta)) dF(\beta)\right].$$
Maximizing $EU^{TU}$ with respect to $s$, and assuming an interior solution, the optimal value of $s$ satisfies
\[
(1 - \pi) U'(s) + \pi F(\tilde{\beta})H'(s + g) = 1. \tag{12}
\]
Observe that, since by $m(\tilde{\beta}) = s + g$, the derivative of $EU^{TU}$ with respect to $\tilde{\beta}$ is zero so that the induced variation in the marginal level of altruism does not appear in (12).

The SOC is given by
\[
(1 - \pi) U''(s) + \pi F(\tilde{\beta})H''(s + g) + \pi f(\tilde{\beta})H'(s + g) \frac{\partial \tilde{\beta}}{\partial (s + g)} < 0,
\]
or, substituting for $\partial \tilde{\beta}/\partial s$
\[
(1 - \pi) U''(s) + \pi H''(s + g) \left[ F(\tilde{\beta}) - \tilde{\beta} f(\tilde{\beta}) \right] < 0,
\]
which is satisfied when $F$ is concave.

Denote the solution to equation (12) by $s^{TU}(g)$. Substituting $s^{TU}(g)$ for $s$ in (12), the resulting relationship holds for all values of $g$. Totally differentiating this relationship while making use of (11) and of the concavity of $F$ yields
\[
\frac{\partial s^{TU}}{\partial g} = -\frac{\pi H''(s + g) \left[ F(\tilde{\beta}) - \tilde{\beta} f(\tilde{\beta}) \right]}{SOC} < 0.
\]
Consequently, we obtain that $s^{TU}(g)$ decreases with $g$. This is not surprising. Savings are useful when the parent is healthy, but also play the role of self-insurance for dependent parents who do not receive formal care. As public LTC becomes available the expected self-insurance benefits associated with $s$ become less important because it only tops up
public care $g$. In other words, individuals can always count on $g$ even when their children fail to deliver; consequently the marginal benefit of $s$ decreases in $g$.

### 3.3 Stage 1: The optimal policy

Let us now determine the levels of $\tau$ and $g$ that maximize $EU^{TU}$, as optimized by the parents in stage 2, subject to the budget constraint

$$\tau w^{T} = \pi g.$$  \hspace{1cm} (13)

Substituting for $\tau$ from (13) into the parents’ optimized value of $EU^{TU}$, $g$ is then chosen to maximize

$$L^{TU} = w^{T} - \pi g - s^{TU}(g) + (1 - \pi) U(s^{TU}(g)) + \pi \left[ \int_{\bar{\beta}}^{\infty} H(m(\beta)) dF(\beta) + F(\bar{\beta}) H(s^{TU}(g) + g) \right].$$

Differentiating $L^{TU}$ with respect to $g$ yields, using the envelope theorem,

$$\frac{\partial L^{TU}}{\partial g} = \pi \left[ F(\bar{\beta}) H'(s^{TU}(g) + g) - 1 \right].$$ \hspace{1cm} (14)

The first term in the RHS of this expression reflects the benefits of an increase in $g$, which is equal to $H'$ for all dependent parents whose children have $\beta < \bar{\beta}$ and to not provide informal care. For the remaining parents, there is full crowding out and no benefit. The second term reflects the cost; since $g$ is given to all dependent parents the expected per unit cost is $\pi$.

Let us evaluate the sign of $\partial L^{TU}/\partial g$ at $g = 0$; it is negative if

$$F(\bar{\beta}) H'(s^{TU}(0)) - 1 \leq 0,$$

where $\bar{\beta} = \bar{\beta}(s(0))$. In this case the solution is given by $g = \tau = 0$ and no insurance should be provided. This condition is satisfied if $F[\bar{\beta}(s(0))]$ is sufficiently small. In that case the probability that individuals receive informal care is so “large” that the benefits of insurance are small and outweighed by its cost in terms of expected crowding out. The *laissez-faire* leaves some individuals (those whose children have $\beta < \bar{\beta}$) without specific LTC benefits (other than self-insurance). This is inefficient, but the $TU$ policy we consider here cannot do better.
A different outcome occurs if

$$F(\tilde{\beta})H'(s^{TU}(0)) - 1 > 0.$$  

In this case, there will be an interior solution for $g$, and $\tau$, characterized by

$$H'(s^{TU}(g^{TU}) + g^{TU}) = \frac{1}{F(\tilde{\beta})} > 1. \quad (15)$$  

Consequently, there is less than full insurance which from (7) would require $H' = 1$. Substituting from (15) into (12), it is also the case that

$$U'(d) = U'(s^{TU}(g^{TU})) = 1,$$

which implies that the parents’ consumption when healthy is at its full information level.

The main results of this section are summarized in the following proposition.

**Proposition 1** Consider a topping up scheme financed by a proportional tax on earnings. Let $s^{TU}(g)$ solve

$$(1 - \pi)U'(s^{TU}(g)) + \pi F(\tilde{\beta})H'(s^{TU}(g) + g) = 1,$$

where

$$\tilde{\beta} = \frac{1}{H'(s^{TU}(g) + g)}.$$  

Two cases may arise:

(i) If $F(\tilde{\beta})H'(s^{TU}(0)) - 1 \leq 0$, public LTC insurance is not effective in supplementing informal care, and $g^{TU} = 0$.

(ii) Otherwise, there is an interior solution balancing insurance benefits against the cost of crowding out informal care. In this case, $g^{TU} > 0$ is defined by

$$H'(s^{TU}(g^{TU}) + g^{TU}) = \frac{1}{F(\tilde{\beta})}.$$  

In either case we have $H' > 1$ so that there is less than full insurance.

### 4 Opting out

In this section we assume that $g$ is exclusive in the sense that it cannot be topped up by $a$ or $s$. The policy is only relevant when $g \geq s$; otherwise, public assistance would be of
no use to the parents. Children’s preferences continue to be given by (2). Under an OO scheme the preferences of children with dependent parents are represented by

$$u = y - a + \beta H(s + a),$$  \hspace{1cm} (16)

if children provide informal care and

$$u = y + \beta H(g),$$  \hspace{1cm} (17)

if they decide not to assist their parents, who then exclusively rely on public LTC insurance.

4.1 Stage 3: The child’s choice

If the child provides care, its level $a^*$ is such that the dependent parents consumption is equal to its laissez-faire level, $m(\beta)$. This follows from the maximization of (16) while making use the definition of $m(\beta)$ in (5). However, children provide care only if this gives them a higher utility than when their parents rely on exclusive government assistance. They thus compare (16) evaluated at $a^*$, with (17), and provide care if

$$\beta[H(m(\beta)) - H(g)] - (m(\beta) - s) > 0. \hspace{1cm} (18)$$

In words, the utility gain from altruism $\beta[H(m(\beta)) - H(g)]$ must exceed the cost of care $a^* = (m(\beta) - s)$. A necessary condition for this inequality to hold is $g < m(\beta)$. The LHS is increasing in $\beta$ for all $g < m(\beta)$, so that for each $g$ and $s$ there exist a $\hat{\beta}(g, s)$ such that all children with $\beta > \hat{\beta}$ provide care, and all children with $\beta \leq \hat{\beta}$ provide no assistance.\textsuperscript{15}

This threshold $\hat{\beta}(g, s)$ is implicitly defined by

$$\hat{\beta} \left[H(m(\hat{\beta})) - H(g)\right] - \left(m(\hat{\beta}) - s\right) = 0.$$

\textsuperscript{15}The derivative of the LHS with respect to $\beta$ is

$$[H(m(\beta)) - H(g)] - \frac{\partial m}{\partial \beta \beta} \left[\beta H'(m(\beta)) - 1\right]. \hspace{1cm} (19)$$

If $\beta \leq \beta_0$, then $\partial m/\partial \beta = 0$. If $\beta > \beta_0$, $\beta H'(m(\beta)) - 1 = 0$. Thus, equation (19) reduces to

$$[H(m(\beta)) - H(g)].$$

which is positive for all $g < m(\beta)$.\textsuperscript{15}
Totally differentiating the expression above yields

\[
\frac{\partial \hat{\beta}}{\partial s} = \frac{1}{H(m(\hat{\beta})) - H(g)} < 0,
\]

and

\[
\frac{\partial \hat{\beta}}{\partial g} = \frac{\hat{\beta} H'(g)}{H(m(\hat{\beta})) - H(g)} > 0.
\]

As in the case with topping up, the threshold of \( \beta \) above which the children provide assistance is increasing in \( g \). However, unlike in the topping up case, this threshold is now decreasing in \( s \). The higher is \( s \), the higher the incentive for the children to provide assistance, otherwise \( s \) would be wasted. In the case of topping up, the opposite was true, and children were less likely to provide assistance if \( s \) was high.

Figure 3 illustrates how the level of consumption of dependent parents depends on the degree of children’s altruism under opting out (solid line). When \( \beta > \hat{\beta} \), parents consume \( m(\beta) \), which is equal to the laissez-faire consumption. If the children’s level of altruism is lower than \( \hat{\beta} \), dependent parents will consume \( g^{OO} \). Unlike in the topping up regime, there is now a discontinuity in the level of \( m \) at \( \hat{\beta} \). This is because under \( OO \) children provide care only if \( m(\beta) \) is sufficiently larger than \( g^{OO} \) to make up for the cost of care.

So far we have implicitly assumed that whenever children are willing to provide care, their parents are prepared to accept it, and thus to forego \( g \). This effectively follows from (18), which requires \( m(\beta) > g \) so that the parent to whom informal care is offered is always better off by opting out of the public LTC system. Intuitively, this does not come as a surprise. Children are altruistic but account for the cost of care, while the latter comes at no cost to parents.

### 4.2 Stage 2: The parent’s choice

The parent’s expected utility function is

\[
EU^{OO} = w(1 - \tau)T - s + (1 - \pi)U(s) + \pi \left[ \int_{0}^{\hat{\beta}} H(g) dF(\beta) + \int_{\hat{\beta}}^{\infty} H(m(\beta)) dF(\beta) \right]
\]

\[
= w(1 - \tau)T - s + (1 - \pi)U(s) + \pi \left[ H(g) F(\hat{\beta}) + \int_{\hat{\beta}}^{\infty} H(m(\beta)) dF(\beta) \right].
\]
Maximizing $EU^{OO}$ with respect to $s$, and assuming an interior solution, the optimal value of $s$ satisfies
\[(1 - \pi) U'(s) - \pi f(\hat{\beta}) \left[ H(m(\hat{\beta})) - H(g) \right] \frac{\partial \hat{\beta}}{\partial s} = 1, \quad (21)\]
Note that the derivatives with respect to $\hat{\beta}$ do not cancel out. Children with altruism $\hat{\beta}$ are indifferent between providing care and not providing it but their parents are not. Recall that children do account for the cost of care, while their parents do not. Substituting $\partial \hat{\beta}/\partial s$ from (20) condition (21) can be written as
\[(1 - \pi) U'(s) + \pi f(\hat{\beta}) = 1. \quad (22)\]
The second term in the LHS of (21) and (22) represents the positive effect of $s$ on the probability that children provide assistance; recall that $\hat{\beta}$ decreases with $s$.

Comparing this expression with (12), we find that for a given level of $g$, savings with $OO$ may be higher or lower than the savings with $TU$. On the one hand, $OO$ reduces the incentives to save, since savings are useful to parents only when dependence does not occur (i.e., with probability $1 - \pi$). On the other hand, savings increase the probability that children provide assistance. Since parents are always better off under family assistance than under public assistance ($H(m(\beta)) - H(g) > 0$ for all $\beta > \hat{\beta}$), this enhances the incentives to save under $OO$.\(^{16}\)

\[16\text{The SOC}\]
\[(1 - \pi) U''(s) + \pi f'(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial s} < 0, \quad (23)\]
is assumed to be satisfied. Now the marginal benefit of savings via the induced increase in family care may increase or decrease in $s$, depending on the slope of the density function $f(\beta)$. Unlike for the SOCs
Denote the solution to equation (22) by \( s^{OO}(g) \). Substituting \( s^{OO}(g) \) for \( s \) in (22), the resulting relationship holds for all values of \( g > s \). Totally differentiating equation (22) yields

\[
\frac{\partial s^{OO}}{\partial g} = -\frac{\pi f'(\hat{\beta})}{SOC} < 0,
\]

(24)

where we use \( f' = F'' < 0 \) which follows from the concavity of \( F \). Consequently, like under \( TU \), \( s^{OO}(g) \) decreases with \( g \). Once again this is due to the fact that the expected self-insurance benefits provided by saving decrease as \( g \) increases. Since \( g \) cannot be topped up, savings do not provide any benefits for parents who receive \( g \). However, an increase in the level of \( g \) affects the probability that informal care is received and this effect is measured by the numerator of (24). As \( g \) increases, the marginal effect of \( s \) on the probability of informal care decreases (due to the concavity of \( F \)), which explains the negative sign of \( \partial s^{OO}/\partial g \).

We now turn to the government’s problem which represents stage 1 of our game. Since we consider a subgame perfect equilibrium in which \( s \) is determined by \( g \), the marginal level of altruism \( \hat{\beta}(g,s) \) now becomes solely a function of \( g \). Therefore we define

\[
\hat{\beta}(g) = \hat{\beta}[g, s^{OO}(g)].
\]

For future reference observe that

\[
\frac{\partial \hat{\beta}}{\partial g} = \frac{\partial \hat{\beta}}{\partial g} + \frac{\partial \hat{\beta}}{\partial s} \frac{\partial s^{OO}}{\partial g} = \frac{\hat{\beta}H'(g)}{H(m(\hat{\beta})) - H(g)} - \frac{\partial s^{OO}}{\partial g} \frac{1}{H(m(\hat{\beta})) - H(g)}.
\]

(25)

This expression accounts for the direct effect of \( g \) and for its indirect impact via the induced variation in \( s \).

### 4.3 Stage 1: The optimal policy

The government’s budget constraint is now given by

\[
\tau \omega T = \pi F(\hat{\beta})(g - s^{OO}(g)).
\]

It differs from (13), its counterpart in the \( TU \) case, in two ways. First, \( g \) is only offered to parents who do not receive informal care, that is a share \( F(\hat{\beta}) \) of the dependent elderly. Considered above, concavity of \( F \) is not sufficient. Quite the opposite: with a concave distribution we have \( f' = F'' < 0 \) so that the second term of (23) is positive.
Second, since \( g \) is exclusive, parents who take up the benefit have to forego their saving. In other words, only \( g - s^{OO}(g) \) has to be financed. Substituting this budget constraint into the parents’ optimized value of \( EU^{OO} \), we are left with choosing \( g \) to maximize

\[
L^{OO} \equiv wT - \pi F(\beta) \big[ g - s^{OO}(g) \big] - s^{OO}(g) + (1 - \pi) U \big[ s^{OO}(g) \big] + \pi \left[ \int_{\beta}^{\infty} H(m(\beta))dF(\beta) + F(\beta)H(g) \right]
\]

Differentiating \( L^{OO} \) with respect to \( g \) yields, using the envelope theorem\(^{17}\)

\[
\frac{\partial L^{OO}}{\partial g} = \pi \left( A - \pi \frac{\partial s^{OO}}{\partial g} \right) - \frac{\partial \hat{\beta}}{\partial g} \frac{F(\beta)H'(g)}{A} - \frac{\partial \hat{\beta}}{\partial g} \frac{f(\beta)\left( H(m(\beta)) - H(g) \right) - (g - s^{OO})f(\beta)\frac{\partial \hat{\beta}}{\partial g}}{B}
\]

This expression shows that an increase in \( g \) has three different effects, labeled \( A, B \) and \( C \).

Term \( A \) measures the expected insurance benefits that \( g \) provides to parents who receive no informal care. The public LTC insurance benefit also affects informal care at the extensive margin: because it increases \( \hat{\beta} \), it reduces the range of altruism parameters for which care is provided. The cost of this adjustment is measured by term \( B \). Finally, \( C \) expresses the impact of an increase in \( g \) on first period consumption. It accounts for the induced adjustments in \( s \) and \( \hat{\beta} \).

Comparing expression (26) to its counterpart in the \( TU \) case, (14), we can see that the first term (insurance benefits) is similar (except that \( \tilde{\beta} \) is replaced by \( \hat{\beta} \)). The term \( C \) (cost) is equal to 1 in the \( TU \) case. Finally term \( B \) is absent in the \( TU \) case because the extensive margin crowding out via \( \tilde{\beta} \) has no first-order effect on parents’ utility.

Substituting from (20) and (25) and rearranging yields

\[
\frac{\partial L^{OO}}{\partial g} = \pi \left[ \left( f(\hat{\beta}) - \frac{g - s^{OO}}{H(m(\beta)) - H(g)} \right) H'(g) - F(\hat{\beta}) \right] + \pi \frac{\partial s^{OO}}{\partial g} \left( f(\hat{\beta}) \left( \frac{g - s^{OO}}{H(m(\beta)) - H(g)} \right) + F(\hat{\beta}) \right).
\]

\(^{17}\)The derivative of the parent’s objective with respect to \( s \) is zero. Consequently the terms pertaining to the induced variation of \( s \), including \( \partial \hat{\beta}/\partial g \) vanish for the parent’s objective but \( \text{not} \) for the budget constraint. This explains why we have \( \partial \hat{\beta}/\partial g \) in term \( B \) but \( \partial \hat{\beta}/\partial g \) in term \( C \).
An interior solution is then characterized by

\[
F(\hat{\beta}) - f(\hat{\beta})\hat{\beta}\left(1 + \frac{g^{OO} - s^{OO}}{H(m(\hat{\beta})) - H(g^{OO})}\right)H'(g^{OO})
\]

\[
= F(\hat{\beta})\left(1 - \frac{\partial s^{OO}}{\partial g}\right) - \frac{\partial s^{OO}}{\partial g} f(\hat{\beta})\left(\frac{g^{OO} - s^{OO}}{H(m(\hat{\beta})) - H(g^{OO})}\right).
\]

Since \(\frac{\partial s^{OO}}{\partial g} < 0\), the RHS of this expression is larger than \(F(\hat{\beta})\) while the term in brackets on the LHS is smaller than \(F(\hat{\beta})\). Consequently, we have \(H'(g) > 1\), implying that under an \(OO\) policy there is less than full insurance for opting-in dependent parents.

The main results of this section are summarized in the following proposition.

**Proposition 2** Consider an opting out scheme financed by a proportional tax on earnings. Let \(s^{OO}(g)\) solve

\[
(1 - \pi)U'(s) + \pi f(\hat{\beta}) = 1.
\]

where the level of altruism of the marginal child \(\hat{\beta}\) is defined by

\[
\hat{\beta}\left[H(m(\hat{\beta})) - H(g)\right] - \left(m(\hat{\beta}) - s^{OO}(g)\right) = 0.
\]

Two cases may arise:

(i) If expression (26) ≤ 0 for \(g = s\), it is not desirable to provide on an exclusive basis a level of \(g\) sufficiently large to be taken up. Then, \(g^{OO} = 0\) or equivalently \(g^{OO} = s^{OO}(g^{OO})\).

(ii) Otherwise, the solution is interior and defined by

\[
\begin{aligned}
&\frac{F(\hat{\beta})H'(g)}{A} - \frac{f(\hat{\beta})\left[H(m(\hat{\beta})) - H(g)\right]}{B} - \frac{F(\hat{\beta})\left(1 - \frac{\partial s^{OO}}{\partial g}\right)}{C} - \left(g - s^{OO}\right)f(\hat{\beta})\frac{\partial \hat{\beta}}{\partial g} = 0,
\end{aligned}
\]

balancing expected insurance benefits, \(A\), against the cost of the induced crowding out at the extensive margin, \(B\), and the budgetary cost \(C\) which translates into a reduction in first period consumption.

In either case we have \(H' > 1\) so that there is less than full insurance.

5 Topping up vs Opting out

The previous sections have shown that under both policies \(g\) will crowd out informal care. In the case of \(TU\) the crowding out occurs both at the intensive and the extensive margins.
At the intensive margin, for all parents who receive care informal care is crowded out by \( g \) on a one by one basis. Crowding out occurs also at the extensive margin, but since the informal care provided by the marginal child \( \beta \) is equal to zero, this has no first-order impact on their parents’ utility. In the case of \( OO \), there is no crowding out at the intensive margin but the crowding out at the extensive margin does have a first-order effect on parents’ utilities. The parents of the marginal children \( \hat{\beta} \) are strictly better off when they receive informal care.

The precise comparison of the \( TU \) and \( OO \) policies is not trivial. To understand the tradeoffs that are involved, we now construct a sufficient condition for \( OO \) to yield a higher welfare than \( TU \). The following proposition is established in Appendix A.

**Proposition 3** Consider the optimal \( TU \) scheme \( g^{TU} \) with saving \( s^{TU} \) and an optimal \( OO \) scheme \( g^{OO} \) provided on an exclusive basis with savings \( s^{OO} \). The \( OO \) scheme dominates if

\[
\pi(1 - F(\hat{\beta}_{TU}))g^{TU} - \pi \int_{\tilde{\beta}}^{(\hat{\beta}_{TV})} [H(m(\beta)) - H(g^{TU} + s^{TU})]dF(\beta) \geq 0.
\]

where \( \hat{\beta}_{TU} = \hat{\beta}(g^{TU} + s^{TU}, s^{TU}) \).

The idea behind this argument is to start from an optimal policy under \( TU \) and then provide a condition under which its replication under \( OO \) is welfare improving. The first term in this expression measures the benefits of switching to \( OO \) while keeping the transfer per beneficiary and savings constant, i.e., setting \( \overline{g}^{OO} = s^{TU} + g^{TU} \). Under \( OO \) we do not have to pay \( g^{TU} \) for the individuals who receive informal care. We can also show that \( \hat{\beta}(g^{TU} + s^{TU}, s^{TU}) > \hat{\beta}(g^{TU}, s^{TU}) \). Since \( m(\beta) > g^{TU} + s^{TU} \) for all \( \beta > \tilde{\beta} \), the second term is negative and measures the cost of switching to \( OO \): individuals in the interval \( [\tilde{\beta}, \hat{\beta}] \) loose from a switch to \( OO \) since they receive aid under \( TU \) but not under \( OO \). Roughly speaking \( OO \) dominates if the share of children with sufficiently large degrees of altruism is large enough. This makes sense: it is for this population that the intensive margin crowding out induced by the \( TU \) policy can be avoided by switching to \( OO \).

This tradeoff is illustrated in Figure 4. For a given \( s = s^{TU} \) and for \( \overline{g}^{OO} = s^{TU} + g^{TU} \), the gray line represents consumption in case of dependence under the \( TU \) regime. The solid black line represents consumption of dependent parents in the \( OO \) regime. As \( \hat{\beta} > \tilde{\beta} \),
area A is the “expected” loss in consumption of dependent parents when the insurance regime switches from TU to OO. Area B represents the savings obtained by switching to OO, which depends on the level of public insurance, as well as on the number of dependent parents receiving family help under OO. The optimal regime will depend on the respective sizes of the two areas. The comparison crucially hinges on the distribution of the altruism parameter, \( F(\beta) \) and on the degree of concavity of the utility function \( H(m) \).

Figure 4: Topping up vs Opting out

\[
g_{TU} + s_{TU} = g_{OO} = g_{TU}
\]

6 Private insurance

So far we have ignored private insurance. Assume now that parents can purchase private insurance \( i \) at an actuarially fair premium \( \pi i \). Furthermore, suppose that private insurance companies cannot enforce an OO contract, which prevents children from helping their parents and forces parents to give away their savings if insurance benefits are claimed.

We show in the Appendix B that in the TU regime, public LTC insurance is a perfect substitute to fair private insurance. Consequently, when fair insurance is available the solution described in Section 3 can be achieved without public intervention. This does not come as a surprise: under TU, the government is not more efficient than a perfectly competitive insurance market. Put differently, there is nothing a public insurer can do that markets cannot also accomplish; public and private insurance are equivalent.

\[\text{18This argument is purely illustrative of the tradoff that is involved. However, the areas cannot directly be compared. First, the area } B \text{ does not account for the distribution of } \beta. \text{ To obtain the effective cost savings one has to multiply area } B \text{ by } [1 - F(\bar{\beta})]. \text{ Second, area } A \text{ represents the loss in consumption, and not in utility. Furthermore, the sum is not weighted by the density.}\]

22
This is no longer true under $OO$, where public insurance brings about the possibility of preventing children’s help and of collecting savings of opting-in dependent parents. Then there may be a role for public intervention.

Since $TU$ public insurance and fair private insurance are equivalent, supplementing private insurance by an $OO$ policy is effectively a special case of the mixed policies studies in the next section. Consequently, we shall not formally study such a policy at this point. Instead we shall return to it in the following section.

7 Mixed policies: opting out and topping up

We now consider the case where the government has two instruments: a transfer to dependent parents that are taken care of by their children, and a transfer to dependent parents whose children fail to provide assistance. The first transfer, $g_{TM}$ can be enjoyed on top of savings and informal care. The second transfer, $g_{OM}$ is exclusive. One can think about the former as monetary or formal care provided at home, and about the latter as nursing home care.

We have shown above that $TU$ is equivalent to fair private insurance. However, this does not imply that the mixed regime we consider is equivalent to the case where an $OO$ policy is implemented in addition to fair private insurance purchased by the parents. In a mixed regime, both $TU$ and $OO$ transfers are chosen simultaneously by the social planner, while private protection is chosen by parents after the public policy is announced. Consequently, the mixed policy (weakly) dominates a combination of fair private insurance and $OO$.

7.1 Stage 3: The child’s choice

If the child decides to provide care, the optimal amount of family assistance $a^*$ is such that the dependent parents consumption is equal to its laissez-faire level, $m(\beta)$. However, children provide assistance only if this gives them a higher utility than exclusive government assistance. Thus, there exist a $\tilde{\beta}(g_{OM}, g_{TM}, s)$ such that all parents with children whose $\beta > \tilde{\beta}$ opt out, while parents with children whose $\beta \leq \tilde{\beta}$ receive no assistance and
This threshold $\hat{\beta}(g_{OM}^{OO}, g_{TM}^{TU}, s)$ is defined by

$$\hat{\beta} \left[ H(m(\hat{\beta})) - H(g_{OM}^{OO}) \right] - (m(\hat{\beta}) - s - g_{TM}^{TU}) = 0.$$ 

Observe that

$$\frac{\partial \hat{\beta}}{\partial s} = \frac{1}{H(m(\hat{\beta})) - H(g_{OM}^{OO})} < 0,$$  

(27)

and

$$\frac{\partial \hat{\beta}}{\partial g_{OM}^{OO}} = \frac{\hat{\beta} H'(g_{OM}^{OO})}{H(m(\hat{\beta})) - H(g_{OM}^{OO})} > 0.$$  

(28)

Consequently, $g_{OM}^{OO}$ makes family help less likely, while the transfer $g_{TM}^{TU}$ provides incentives for children to provide some help. This is due to the fact that the cost of ensuring a consumption level $m(\beta)$ to parents decreases in $g_{TM}^{TU}$. This property is important for understanding the respective roles played by the two policies and to comprehend why it may be optimal to combine them. It is also brought out by Figure 5 below. It shows that increasing $g_{OM}^{OO}$ makes the provision of informal care less attractive to children. On the other hand, increasing $g_{TM}^{TU}$ makes the provision of care more appealing, as any given level of total care $m$ can be achieved at a lower cost to children.

### 7.2 Stage 2: The parent’s choice

The parent’s expected utility function is

$$EU^M = w(1 - \tau) \bar{T} - s + (1 - \pi) U(s) + \pi \left[ H(g_{OM}^{OO}) F(\hat{\beta}) + \int_{\beta}^{\infty} H(m(\beta)) dF(\beta) \right].$$

The derivative of $EU$ with respect to $s$ is

$$(1 - \pi) U'(s) - \pi f(\hat{\beta}) \left[ H(m(\hat{\beta})) - H(g_{OM}^{OO}) \right] \frac{\partial \hat{\beta}}{\partial s} - 1.$$  

(29)

Assuming an interior solution and using (27) and (28) the following expression for the optimal level of savings yields

$$(1 - \pi) U'(s) + \pi f(\hat{\beta}) = 1.$$  

(30)

We assume that the second-order condition is satisfied

$$(1 - \pi) U''(s) - \pi \frac{f'(\hat{\beta})}{H(m(\hat{\beta})) - H(g_{OM}^{OO})} < 0.$$  

We assume that the second-order condition is satisfied

$$(1 - \pi) U''(s) - \pi \frac{f'(\hat{\beta})}{H(m(\hat{\beta})) - H(g_{OM}^{OO})} < 0.$$  

24
Let us denote by $s^M(g^O_M,g^T_M)$ the optimal level of savings as a function of the government transfers. Once again we can then define the optimal level of altruism solely as a function of the policy instruments: $\hat{\beta}(g^O_M,g^T_M) = \hat{\beta}(g^O_M,g^T_M,s^M(g^O_M,g^T_M))$. Observe that $s^M(g^O_M,g^T_M)$ and $\hat{\beta}(g^O_M,g^T_M)$ are jointly defined by the system of equations (29)–(30). In Appendix C we show that

$$\frac{\partial s^M}{\partial g^O_M} < 0, \quad \frac{\partial s^M}{\partial g^T_M} > 0, \quad \frac{\partial \hat{\beta}}{\partial g^O_M} > 0, \quad \text{and} \quad \frac{\partial \hat{\beta}}{\partial g^T_M} < 0.$$ 

Observe that the effects of an increase in $g^O_M$ are similar to the ones obtained under the pure $OO$ scenario. Conversely, an increase in $g^T_M$ now has opposite effects than under the pure $TU$ scenario. In the pure $TU$ scenario the public transfer is received by all dependent parents, and crowds out informal care at the intensive and the extensive margins. As a consequence, an increase in the transfer reduces savings and increases the threshold above which children provide help. In the mixed regime, however, the $TU$ transfer is only received by parents that opt out from the exclusive public provision. Consequently, an increase in this transfer makes it more attractive to opt out and rely on family care. This explains why $\hat{\beta}$ decreases.

Since $F(\beta)$ is concave, parents also anticipate that the positive effect of $s$ on $\hat{\beta}$ increases as $g^T_M$ increases. Consequently, $g^T_M$ also has a positive effect on savings. In Appendix C we also show that

$$\hat{\beta} H'(g^O_M) = \frac{\partial s^M}{\partial g^O_M} \frac{d g^T_M}{d g^O_M} \bigg|_{s} = \frac{\partial \hat{\beta}}{\partial g^O_M} \frac{d g^T_M}{d g^O_M} \bigg|_{\hat{\beta}}.$$

This equation gives the “marginal rates of substitution” between $g^O_M$ and $g^T_M$, for a given level of $s$ and a given level of $\hat{\beta}$ and shows that these two expressions are equal. A marginal increase in $g^O_M$ has to be compensated by an increase $\hat{\beta} H'(g^O_M)$ in $g^T_M$ to ensure that $\hat{\beta}$ and $s$ are held constant. Consequently, any effect of $g^O_M$ on the individual behaviors can be compensated by an appropriate increase in $g^T_M$. The intuition for this result is illustrated in Figure 5. The solid line represents the consumption of dependent parents in the mixed regime, which is not directly affected by $g^T_M$. An increase in $g^T_M$ affects consumption only by decreasing $\hat{\beta}$. As $g^O_M$ increases, the utility of the marginal children (with altruism $\hat{\beta}$) when they provide no care increases by $\hat{\beta} H'(g^O_M)$. So when $g^T_M$ increases by this amount, these children remain indifferent between providing and not
providing care. Since savings affect \(m\) only indirectly through \(\beta\), \(s\) also remains unchanged as long as \(dg_M^{TU}/dg_M^{OO} = \beta H'(g_M^{OO})\).

Figure 5: Mixed regime. Consumption of dependent parents as a function of children’s altruism

![Graph showing consumption of dependent parents](image)

7.3 Stage 1: The optimal policy

The government chooses \(g^{TU}\) and \(g^{OO}\) to maximize

\[
\mathcal{L}^M \equiv wT - \pi F(\hat{\beta})(g_M^{OO} - s^M) - \pi \left[1 - F(\hat{\beta})\right] g_M^{TU} - s^M + (1 - \pi) U(s^M) + \pi \left[\int_0^\infty H(m(\beta))dF(\beta) + F(\hat{\beta})H(g_M^{OO})\right].
\]

Differentiating \(\mathcal{L}\) with respect to \(g^{TU}\) and \(g^{OO}\), and using the envelope theorem, we obtain

\[
\frac{\partial \mathcal{L}^M}{\partial g_M^{OO}} = \pi \left[F(\hat{\beta}) \left(H'(g_M^{OO}) - 1 + \frac{\partial s^M}{\partial g_M^{OO}}\right) - f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g_M^{OO}} (g_M^{OO} - s^M - g_M^{TU}) - f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g_M^{OO}} \Delta H\right],
\]

and

\[
\frac{\partial \mathcal{L}^M}{\partial g_M^{TU}} = \pi \left[F(\hat{\beta}) \left(1 + \frac{\partial s^M}{\partial g_M^{TU}}\right) - 1 - f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g_M^{TU}} (g_M^{OO} - s^M - g_M^{TU}) - f(\hat{\beta}) \frac{\partial \hat{\beta}}{\partial g_M^{TU}} \Delta H\right].
\]

where \(\Delta H = H(m(\beta)) - H(g_M^{OO})\). Using the conditions in (31) and substituting (33) in (32) we obtain the following condition for an interior solution

\[ F(\hat{\beta}) \left(H'(g_M^{OO}) - 1 - \beta H'(g_M^{OO}) \left(1 - F(\hat{\beta})\right) \right) = 0. \]

This expression is intuitive. It shows the tradeoff between \(g_M^{OO}\) and \(g_M^{TU}\) for given levels of \(\hat{\beta}\) and \(s\). The optimal policy must satisfy the following (necessary) condition: welfare
cannot be increased by a “compensated” variation in $g_M^{OО}$ and $g_M^{TU}$ that leaves $\hat{\beta}$ and $s$ unchanged, that is a variation such that $dg_M^{TU} = \hat{\beta}H' (g_M^{OО}) \, dg_M^{OО}$; see equation (C6).

The first term in (34) represents the net marginal benefit of $g_M^{OО}$ for $\hat{\beta}$ and $s$ given. An increase in $g_M^{OО}$ entails an increase in the utility for dependent parents who do not receive family help, and a marginal cost equal to 1. In the second term, $(1 - F(\hat{\beta}))$ represents the cost of increasing $g_M^{TU}$, while $\hat{\beta}H' (g_M^{OО})$ represents the increase in $g_M^{TU}$ ensuring that $\hat{\beta}$ and $s$ are held constant as $g_M^{OО}$ increases. When choosing $g_M^{OО}$, the social planner takes into account the need to provide insurance to dependent parents with no family help, but also the fact that the insurance provided to parents who receive informal care, $g_M^{TU}$ will have to adjust in order to keep $\hat{\beta}$ and $s$ constant. Put differently, parents who do not receive informal care because $g_M^{OО}$ is available should be compensated by a sufficiently large $g_M^{TU}$ to neutralize the crowding out of savings and family help. The advantage of a mixed policy is that it makes this compensation possible.

Condition (34) also implies that $H'(g_M^{OО}) > 1$ as long as $1 - F(\hat{\beta}) > 1$. In words, the optimal mixed regime implies less than full insurance for the parents who do not receive family help, except in the case where no child provides any help (i.e. $\hat{\beta} = \bar{\beta}$).

Finally, the tradeoff we just described is relevant only when both instruments are set by the government. When $g_M^{TU}$ is replaced by private insurance, individual coverage is controlled only indirectly. The compensated variation considered in (34) is then no longer feasible.

The main results of this section are summarized in the following proposition.

Proposition 4 Consider a mixed scheme where parents can choose between a transfer $g_M^{TU}$ that can be topped up and a transfer $g_M^{OО}$ which is provided on an exclusive basis. Redefine the marginal child $\hat{\beta}$ such that

$$\hat{\beta} \left[ H(m(\hat{\beta})) - H(g_M^{OО}) \right] - \left( m(\hat{\beta}) - s - g_M^{TU} \right) = 0.$$

(i) Assuming an interior solution, the optimal mixed policy is characterized by

$$F(\hat{\beta}) \left( H'(g_M^{OО}) - 1 \right) - \hat{\beta}H' (g_M^{OО}) \left( 1 - F(\hat{\beta}) \right) = 0,$$

which shows the tradeoff between $g_M^{OО}$ and $g_M^{TU}$ for given levels of $\hat{\beta}$ and $s$. The optimal policy must be such that welfare cannot be increased by a “compensated” variation in $g_M^{OО}$ and
that leaves $\beta$ and $s$ unchanged, that is a variation such that $dg_M^{TU} = \beta H'(g_M^{OO}) dg_M^{OO}$.

Consequently, the policy is designed to provide insurance, via $g_M^{OO}$, to dependent parents who do not receive informal care, while $g_M^{TU}$ will be set to keep $\beta$ and $s$ constant.

(ii) The policy implies less than full insurance.

8 Conclusion

This paper has studied the role of social insurance programs in a world in which family assistance is uncertain. It has considered the behavior and welfare of a single generation of “parents” over their life cycle. It has considered social LTC under $TU$ and $OO$ out regimes.

With $TU$, crowding out occurs both at the intensive and the extensive margins (level of care and share of children who provide care). With $OO$ there is no crowding out at the intensive margin, but the one at the extensive margin may be exacerbated. We have provided a sufficient condition under which $OO$ dominates $TU$. Roughly speaking this requires that the share of children with sufficiently large degrees of altruism is large enough. This makes sense: its for this population that the intensive margin crowding out induced by the $TU$ policy can be avoided by switching to $OO$.

Finally, we have considered a policy combining financial aid on a $TU$ basis with public $OO$ care provision. We have shown that the policies interact in a nontrivial way. When combined in an appropriate way the policies can effectively be used to neutralize their respective distortions. For instance, variations in the policies can be designed so that the marginal level of altruism (above which children provide care) and savings are not affected. Consequently the mixed policy may be an effective way to provide LTC insurance coverage, even when none of the policies is effective when used as sole instrument.

Our results highlight a tradeoff that can inform policy makers considering different schemes for financing long term care. However, the analysis lies on some simplifying assumptions. First, we assume that parents cannot influence the amount of family help, for instance through strategic bequests. Second, we assume that the social planner takes into account only the utility of the parents’ generation. Relaxing these assumptions is on our research agenda. These investigations would require changes in the setup which are
too drastic to be included in the current paper.
Appendix

A Appendix: Topping up vs Opting out

We start from the optimal policy under TU and examine under which conditions it can be replicated under OO.

Consider the optimal policy under TU, \(g_{TU}\), which yields \(s_{TU}\) and a level of welfare defined by

\[ EU_{TU} = wT - \pi g_{TU} - s_{TU} + (1 - \pi) U(s_{TU}) + \pi \left[ \int_\beta^\infty H(m(\beta)) dF(\beta) + F(\tilde{\beta})H(s_{TU} + g_{TU}) \right]. \]  

(A1)

Let us replace this policy by an OO policy with \(g_{OO} = g_{TU} + s_{TU}\). We then have

\[ EU_{OO} \geq wT - \pi F(\hat{\beta}(g_{OO}))(g_{OO} - s_{TU}) - s_{TU} + (1 - \pi) U(s_{TU}) + \pi \left[ \int_{\hat{\beta}(g_{OO})}^\infty H(m(\beta)) dF(\beta) + F(\tilde{\beta}(g_{OO}))H(g_{OO}) \right] \],

where the inequality sign appears because neither \(g_{OO} = g_{TU} + s_{TU}\) nor \(s_{TU}\) are in general the optimal levels of insurance and savings under OO. This can be rewritten as

\[ EU_{OO} \geq wT - \pi F(\hat{\beta}(g_{OO}))(g_{TU} + s_{TU}) - s_{TU} + (1 - \pi) U(s_{TU}) + \pi \left[ \int_{\hat{\beta}(g_{OO})}^\infty H(m(\beta)) dF(\beta) + F(\tilde{\beta}(g_{OO}))H(g_{OO}) \right]. \]  

(A2)

Observe that when \(g_{OO} = g_{TU} + s_{TU}\) and \(s = s_{TU}\) we have \(\hat{\beta} > \tilde{\beta}\). To see this, evaluate (18) at \(\tilde{\beta}\) which yields

\[ \tilde{\beta} \left[ H(m(\tilde{\beta})) - H\left(g_{OO}\right) \right] - \left( m(\tilde{\beta}) - s_{TU} \right). \]

From the definition of \(\tilde{\beta}\) we have that \(m(\tilde{\beta}) = g_{TU} + s_{TU}\), so that this equation can be rewritten as

\[ \tilde{\beta} \left[ H(g_{TU} + s_{TU}) - H\left(g_{TU} + s_{TU}\right) \right] - \left( g_{TU} + s_{TU} - s_{TU} \right) = -g_{TU} < 0. \]

In words, children with \(\beta = \tilde{\beta}\) will not provide aid under OO so that we must have \(\hat{\beta}_{TU} = \tilde{\beta}\).
\( \hat{\beta}(g^{TU} + s^{TU}, s^{TU}) > \hat{\beta} \). Then, combining (A1) and (A2) implies that \( EU^{OO} \geq EU^{TU} \) if

\[
\pi(1 - F(\hat{\beta}^{TU}))g^{TU} - \pi \int_{\hat{\beta}}^{\tilde{\beta}} H(m(\beta)) dF(\beta) + \pi[F(\hat{\beta}^{TU}) - F(\hat{\beta})]H(g^{TU} + s^{TU}) = \\
\pi(1 - F(\hat{\beta}^{TU}))g^{TU} - \pi \int_{\hat{\beta}}^{\tilde{\beta}} [H(m(\beta)) - H(g^{TU} + s^{TU})]dF(\beta) \geq 0.
\]

**B Appendix: Topping up and private insurance**

In the case with \( TU \) and actuarially fair private insurance, individuals can purchase an insurance coverage \( i \), to be received in case of dependence. The fair premium is \( \pi_i \).

The first-order condition of the children’s problem with respect to \( a \) is, assuming an interior solution,

\[-1 + \beta H'(s + g + i + a) = 0.\]

Define \( \tilde{\beta}(s + g + i) \) such that

\[1 = \tilde{\beta} H'(s + g + i)\]  

(B1)

If \( \beta \geq \tilde{\beta} \), the consumption of dependent parents \( m(\beta) \) is exactly the same as in the laissez-faire. When \( \beta < \tilde{\beta} \), \( a^* = 0 \) and \( m = s + g + i \). Finally, observe that

\[\frac{\partial \tilde{\beta}}{\partial(s + g + i)} = -\frac{\tilde{\beta} H''}{H'} > 0.\]

The problem of the parents is to maximize their expected utility with respect to \( s \) and \( i \), and assuming an interior solution (i.e. \( i > 0 \)), the optimal value of \( s \) and \( i \) satisfy, respectively

\[(1 - \pi)U'(s) + \pi F(\tilde{\beta})H'(s + g + i) = 1, \]  

(B2)

and

\[F(\tilde{\beta})H'(s + g + i) = 1, \]  

(B3)

which implies

\[U'(s) = F(\tilde{\beta})H'(s + g + i) = 1. \]  

(B4)

Consequently, \( s \) does not depend on the level of public LTC insurance, and \( \partial i / \partial g = -1. \)
In stage 1, the government chooses \( g \) to maximize

\[
\mathcal{L} \equiv wT - \pi g - s - \pi i + (1 - \pi) U(s) + \\
\pi \left[ \int_{\hat{\beta}}^{\infty} H(m(\beta)) \, dF(\beta) + F(\hat{\beta}) H(s + g + i) \right].
\] (B5)

Differentiating \( \mathcal{L} \) with respect to \( g \) yields, using the envelope theorem,

\[
\frac{\partial \mathcal{L}}{\partial g} = \pi \left[ F(\hat{\beta}) H'(s + g + i(g)) - 1 \right],
\] (B6)

which, under (B3), is equal to zero for all \( g \) such that \( F(\hat{\beta}) H'(s + g) \geq 1 \), and negative otherwise, when \( g \) is so large that the constraint that \( i \geq 0 \) becomes binding (so that \( i = 0 \)). Then, a fair LTC private insurance is available can fully replace a public TU regime.

### C Appendix: Mixed Regime

Differentiating (29)–(30) and solving by using Cramer’s rule yields

\[
\frac{\partial s^M}{\partial g^{OO}} = \frac{-\pi f'(\hat{\beta}) \hat{\beta} H'(g^{OO})}{(1 - \pi) U'(s) \Delta H - \pi f'(\hat{\beta})} < 0,
\] (C1)

\[
\frac{\partial s^M}{\partial g^{TU}} = \frac{\pi f'(\hat{\beta})}{(1 - \pi) U'(s) \Delta H - \pi f'(\hat{\beta})} > 0,
\] (C2)

\[
\frac{\partial \hat{\beta}}{\partial g^{OO}} = \frac{(1 - \pi) U'(s) \hat{\beta} H'(g^{OO})}{(1 - \pi) U'(s) \Delta H - \pi f'(\hat{\beta})} > 0,
\] (C3)

\[
\frac{\partial \hat{\beta}}{\partial g^{TU}} = \frac{-(1 - \pi) U'(s)}{(1 - \pi) U'(s) \Delta H - \pi f'(\hat{\beta})} < 0,
\] (C4)

where \( \Delta H = H(m(\hat{\beta})) - H(g^{OO}) \).

Conditions (C1)–(C4) imply

\[
\frac{\partial s^M}{\partial g^{OO}} = -\hat{\beta} H'(g^{OO}) \frac{\partial s^M}{\partial g^{TU}}, \quad \frac{\partial \hat{\beta}}{\partial g^{OO}} = -\hat{\beta} H'(g^{OO}) \frac{\partial \hat{\beta}}{\partial g^{TU}}, \quad \frac{\partial \hat{\beta}}{\partial g^{OO}} = -\hat{\beta} H'(g^{OO}) \frac{\partial \hat{\beta}}{\partial g^{TU}}.
\] (C5)

Combining these expressions we obtain

\[
\hat{\beta} H'(g^{OO}) = -\frac{\partial s^M}{\partial g^{TU}} \bigg|_s = -\frac{\partial \hat{\beta}}{\partial g^{TU}} \bigg|_{\hat{\beta}} = \frac{d g^{TU}_M}{d g^{MO}_M} \bigg|_{\hat{\beta}}.
\] (C6)
References


