Motivating versus Funding∗

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Abstract. We consider a moral hazard problem where the agent’s effort induces monetary costs, and limits on the agent’s resource restrict his capability to exert effort. The optimal contract is, in some cases, a sharing contract and the principal provides the agent with an up-front financial transfer. Moreover, whereas incentives and transfer to the agent are substitutes in the case where he has sufficient wealth, they are complements when his wealth is limited. Also, if the agent can consume some of his wealth at the outset of the contractual arrangement, he can get all the surplus of the relationship.

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1 Introduction

Is it optimal to provide incentives or to fund an agent whose wealth is limited? What is the optimal incentive/funding structure? Providing incentives to an agent who does not have the financial capability that would enable him to reach a desirable outcome, or expanding the financial capability of an agent who faces no incentives, may lead to inefficient economic activity. While this trade-off has surprisingly received limited attention in the literature, the conflict between funding and providing incentives is an essential feature of numerous contractual relationships, as illustrated by the following examples.

Startups and small businesses are often characterized by difficult access to capital markets and long-term growth potential. They mainly rely on venture capitalists to raise funding at early stages of development, which enlarges their development potential.\(^1\) Venture capitalists are concerned that the entrepreneur does not shirk and does not invest in perks at the expense of productive investments (Bergemann and Hege, 1998).\(^2\) In practice, venture capitalists typically provide both funding at the outset of the relationship and incentives to the entrepreneur via shares in the business.

In franchise contracts, franchisees tend to benefit from the brand name, and franchisers are concerned that the actions of franchisees do not deteriorate the reputation of the brand name by, for instance, producing lower quality products to reduce costs (Brickley and Dark, 1987). Franchisees typically pay both a franchise fee, at the onset of the relationship, and a royalty rate. Franchisers thus face a trade-off between providing incentives, via a lower royalty rate, and financial means (a lower franchise fee) to the franchisee, who needs to undertake appropriate investment into the business.

Regarding natural resources, Payment for Environmental Services (PES) have received increasing attention as an incentive based instrument to help improve conservation (see Alix-Garcia and Wolf 2014). Such instruments are contractual arrangements that stipulate conditional payments to resource owners, which are effective if pre-defined conservation targets are achieved. In such situations, it may be difficult and costly to assess whether deforestation occurs because of natural hazards or because of the owner’s will. However, there is a rising concern that liquidity constrained resource owners may not be capable of achieving targets because they face large opportunity costs (Jayachandran, 2013). There is thus a trade-off between providing both incentives, by using a payment at the end of the contract, and financial aid to help poor owners deal with pressing opportunity costs.

Another interesting example relates to the relationship between wealth and cognitive abilities. Many et al. (2013) focus on the case of sugarcane farmers in India, and provide experimental and field evidence on the fact that low levels of wealth may impede cognitive capacities.\(^3\) Within a contractual setting, the corresponding trade-off will lie in providing incentives (via bonus payments) versus financial means (funding to improve the agent’s cognitive capacity).

In our environment, as in standard moral hazard models, the agent takes an unobserved costly action, which produces a stochastic output. The principal provides incentives by paying the agent based on the observed output and by providing him with (or by requiring) an up-

\(^1\)See Metrick and Yasuda 2010 for a survey.
\(^2\)See also Mehta (2004).
\(^3\)Carvalho et al. (2016) do not find such a relationship in the case of a group of low-income US households.
front payment. Unlike in standard models, we consider that the agent’s effort induces monetary costs (as well as non monetary costs, as in the standard setting). The monetary cost can be directly covered by the agent’s choice to undertake financial investments, while the principal can indirectly cover it through financial transfers to the agent. In this context, the agent’s budget constraint restricts the set of feasible actions. Thus, the lower the level of wealth of the agent, including transfer from or to the principal, the lower the maximum effort level that the agent can provide.

The principal and the agent are both risk neutral, so that the only distortion comes from the budget constraint, which limits the agent’s set of feasible actions. When the budget constraint binds, the up-front payment and the effort become rivals. The agent cannot pay a large up-front payment and supply a high effort level.

We show in this setting that funding is sometimes optimal, but it prevents the project owner to get the full return of the project. Indeed, for intermediate levels of the budget constraint, the principal optimally chooses a sharing contract, knowing that the agent will exert a sub-optimal effort level. However, this enables the principal to pay a lower up-front payment to (or to ask for a larger up-front payment) the agent. In such a case, the agent benefits from having a limited budget and gets some positive rents, while he gets no rent in the benchmark case without asymmetric information.

We highlight a major difference between the case where the agent has sufficient wealth, and the case in which his wealth is sufficiently constrained. Indeed, when the agent has sufficient wealth, incentives and monetary transfer to the agent are substitutes, while they vary in the same direction when the agent has sufficient but limited wealth. Specifically, when the agent is sufficiently wealthy, the bonus paid to him increases, while the transfer received from the principal decreases, with an increase in the value of the project. In contrast, when the agent’s wealth is sufficiently low (but not too low, otherwise no contract is signed) the bonus and the transfer either both increase or both decrease as the value of the project increases.

We also show that, if the budget of the agent is endogenous, the agent can get the full surplus of the relationship. Indeed, assuming that the agent can consume some of his wealth before the contract is signed, we show that the agent keeps the minimum level of wealth so that the principal offers him a contract. He does not keep more wealth than this level in order to induce the principal to pay the largest (or to ask for the smallest) feasible up-front payment. All the results described above are shown to depend on the existence of monetary costs: they do not exist in the polar case of strictly non-monetary costs, which is actually consistent with standard models used in Lewis and Sappington (2000a,b, 2001).

Coming back to the real-world examples provided above, this analysis delivers several predictions and policy implications. In the case of venture capital, while our model is stylized, it is, to our knowledge, the first to show that two main features of venture capital (see for instance Kaplan and Strömberg 2003), funding and profit sharing, can endogenously emerge as the optimal contractual arrangement. Indeed, our model predicts that, under limited wealth and moral hazard, it is sometimes optimal that the principal provides funding and retains large stakes in the funded business. The model also predicts that, when the potential profitability of the business increases, the venture capitalist will either provide a large share of funding and retain small stakes in the business, or provide a smaller share of funding and retain large stakes
in the business. Our model has also interesting predictions in the case of franchising. When the wealth constraint of the franchisee is not too severe, the optimal contract includes a fee to be paid by the franchisee and a royalty rate (i.e. a share of the output is given to the principal). Our analysis is consistent with the recent empirical results in Fan et al. (2016), who show that franchising probability increases with an increase in financial means. Other consistent predictions are that franchise fees increase while royalty rates decrease as financial means increase, and that fees and royalty rates either simultaneously increase or decrease with the profitability of the business. In terms of policy implications, our findings also suggest that PES should include up-front payments when agents are financially constrained, which is consistent with Jayachandran (2013). We discuss these empirical implications of our model, and their relevance in the settings of the examples, in more detail in section 6.

Regarding the relationship between wealth and cognitive abilities, one of the applications of the model (in Section 6.4) is related to the problem of aiding an agent to raise his capabilities, that we model as an individual potential. In many situations an agent can improve his potential: for instance, this can be achieved by education or training. Such situations can be accounted for by the present model. In this application, the agent’s capability corresponds to the different levels of effort he can supply. It is measured by the length of the interval starting from no effort to a maximal effort level. The higher this maximal effort level is, the higher is the agent’s capability. Then a financial aid may help the agent to improve his potential in a particular task, by removing other constraints that exist (up-front) at the outset of the contractual relationship, such as consumption constraints, health-related constraints, or technological constraints.

The remainder of the paper proceeds as follows. The related literature is presented in Section 2. The basic model is introduced in Section 3. Its analysis is provided in Section 4. The possibility that the agent consumes part of his wealth at the outset of the contractual relationship is considered and analyzed in Section 5. Four applications of the model related to, respectively, venture capital, franchising, payment for environmental services and the issue of raising individual potential, are developed in Section 6 in order to derive testable predictions and discuss some policy implications. Finally, Section 7 concludes. All proofs are provided in an appendix at the end of the paper.

2 Related Literature

Regarding the related literature, the effect of financial constraints has been analyzed in a variety of settings, but the effect of limited wealth that restricts the agent’s set of feasible actions has not been considered yet. The potential effect of wealth on the agent’s set of feasible actions is conceptually different from the classical notion of limited liability (see Pérez-Castrillo and Macho-Stadler 2016, Laffont and Martimort 2002, and Sappington 1983 for a seminal contribution). A limited liability constraint might either bound the feasible payments from the principal

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4The notion of capability is the object of a large literature, see Dal Bo et al. (2013) for a recent empirical contribution. There are three well-known strands of literature developing theories of capability: the capability approach of development in philosophy and economics (Sen, 1985, 1989, 1999), the social cognitive theory of human behavior in psychology, which introduces five related types of capability and the self efficacy concept (Bandura, 1977, 2001), and the resource based theory of the firm (Wernerfelt, 1984; Conner and Prahalad, 1996) in management sciences.
to the agent (which must lie above an exogenous threshold, see Jewitt et al. 2008, and Poblete and Spulber 2012, for recent related analyzes) or the agent’s ex-post payoff. Compared to the case analyzed here, liability constraints do not restrict the agent’s feasible actions while, in our model, wealth constraints restrict these actions. Thus, the situation analyzed here differs from the limited liability case, because in the present setting the resource constraint makes the maximum effort dependent on the up-front financial transfer, while limited liability constraints do not result in this interplay.

Looking at the closest related contributions, our analysis differs from Lewis and Sappington (2000a,b, 2001) who focus on constraints on the up-front payment, but not on the level of investment of the agent. More specifically, in all these contributions, agents are assumed to make bond payments at the outset of the principal-agent relationship, even though the principal can promise to return some portion of the initial bond in Lewis and Sappington (2001). This differs from the situations we focus on, where it is sometimes optimal for the principal to make a positive up-front transfer to the agent. In Lewis and Sappington (2000b) the authors briefly consider a situation where the principal can provide a productive input.5 This contrasts with the present analysis, where the principal does not provide a productive input but rather financial means (see a related application to venture capital in Section 6.1). Lewis and Sappington (2001) consider an adverse selection problem in which the agent is privately informed about his wealth and ability, and analyze the role of these two fundamentals on the power of the incentive scheme designed by the principal.6 By contrast, we focus on a situation where the agent’s ability is endogenous, and analyze how this influences the cases where the principal offers a contract and the power of the incentive scheme. The present paper contributes to the literature by analyzing the optimal trade-off between aid (enhancing the agent’s ability to supply effort) and incentives (inducing the agent to supply effort). It complements Lewis and Sappington (2000b) as our model contributes to providing further explanation, in the context of venture capital contracts, about why venture capitalists typically maintain a significant ownership stake in the ventures they finance (Sahlman, 1990) and why venture capitalists will not always provide large share of proceeds to capable entrepreneurs. Moreover, it contributes to the question of optimal contracting when both investment policy choices and effort are made privately by the agent. This contrasts with most of the literature on financial contracts, where the investment level is usually assumed to be exogenous (see Bester 1987 and Innes 1990 among other examples).

Laffont and Matoussi (1995) consider a moral hazard problem in which the agent has limited budget for a verifiable input. There is no potential relationship between wealth and effort level, while this relationship is central in our contribution. More specifically, these authors consider two polar cases. In the first one, the agent supplies effort (say labor), there is no investment, and the agent cannot manipulate his resource constraint. In the second one, there is no labor supplied, the agent can make an investment and possibly manipulate the resource constraint, but there is no provision of incentives by the principal. Thus, this contribution cannot consider the optimal mix between aid and incentives, as in the present paper. Che and Gale (2000) study

5In the case of franchise contracts, Mathewson and Winter (1985) analyze double-sided moral hazard problems where both the agent and the principal provide efforts.

6The power of an incentive scheme (see Lazear 2000 among other references) is the rate at which the agent’s payoff increases as the surplus he generates increases. In the present paper, the power of the incentive scheme under which the agent operates is the share of the value of the project the agent receives in case of a success.
selling mechanisms with limited budget under adverse selection and Burkett (2015) consider the case with budget manipulation by the agent (see also Benoit and Krishna 2001). In the present paper, we consider a moral hazard problem and consider the possibility that the agent spends his budget for consumption in anticipation of the project. This leads to the interesting result that an agent may get the full surplus of the relationship.

Finally, our analysis is also related to the literature on aid. Azam and Laffont (2003) study aid contracts and Cordella and Dell’Ariccia (2007) focus on the comparison of budget support and project aid, in a situation in which the principal can (imperfectly) control the receiver inputs but cannot provide incentives based on the output level. As mentioned above, we focus on the optimal mix between aid and incentives in the present analysis.

3 The Model

We consider a principal-agent model in which a principal may contract with an agent in order to complete a project. The budget of the agent is $B$. The principal may choose to propose a contract to the agent or to get his outside option, $\phi^P \geq 0$. The probability of success of the project $p$ is an increasing and concave function of an unobservable effort of the agent $e \geq 0$, that is $p(e)$, which is assumed to be twice continuously differentiable, strictly increasing and strictly concave, $p'(e) > 0$ and $p''(e) < 0$. If the project is a success, the principal gets a positive return $V > 0$. The effort of the agent is not verifiable, thus the contract is only based on the success or failure of the project. The contract specifies both an up-front payment from the principal to the agent, $T$ (which may be negative or positive), and a bonus paid to the agent, $w$, in case of success. If the project fails, the agent receives no bonus. If the agent rejects the contract, he gets $\phi^A \geq 0$ and he consumes his wealth, $B$. If he accepts the contract, the principal pays him the up-front payment $T$, which is possibly a payment from the agent to the principal. However, it cannot exceed the budget of the agent, i.e. $-T \leq B$. Then, the agent chooses an effort level $e \in [0, \bar{e}]$, and incurs monetary cost $K(e)$ and non monetary cost $c(e)$. These costs include the monetary and non-monetary costs of supplying effort level $e$, of being capable of exerting this effort level, as well as the opportunity cost of exerting effort level $e$. The monetary cost $K(e)$ is constrained by the budget of the agent. The budget constraint writes:

$$K(e) \leq B + T. \quad (1)$$

This assumption is consistent with various situations where the agent is wealth constrained: the opportunity cost can be monetary, the effort provided by the agent can actually be an investment, or the agent can invest in training activities in order to improve his potential. These various settings are illustrated by several applications in Section 6. If the project succeeds, the agent receives the bonus, consumes his remaining wealth $(B + T - K(e))$ and incurs the non monetary cost of effort $(c(e))$, then $u^A = w + B + T - K(e) - c(e)$. The principal gets the return of the project, pays the bonus to the agent, and pays the up-front payment, then $u^P = V - w - T$. If the project fails, the agent receives no bonus, but he consumes his remaining wealth $(B + T - K(e))$ and incurs the non monetary cost of effort, then $u^A = B + T - K(e) - c(e)$. The principal gets

\[^7\text{We assume that } \bar{e} \text{ is large enough so that it never constrains the optimal solution.}\]
no return from the project, pays no bonus to the agent, but gets the up-front payment, then \( u^P = -T \). The agent’s expected payoff is then:

\[
Eu^A = p(e) w + B + T - K(e) - c(e),
\]

(2)

and the principal’s expected payoff is:

\[
Eu^P = p(e) (V - w) - T.
\]

(3)

Notice that the outside option of the principal is \( \phi^P \), while the outside option of the agent is \( \phi^A + B \).

**Assumption (A1): The project is not worthwhile if the effort level is zero:**

\[
p(0)V - K(0) - c(0) < \phi^A + \phi_P.
\]

(4)

If the principal does not propose a contract, he gets his outside option, \( \phi^P \). If he chooses to propose a contract to the agent, the principal’s problem is the following:

\[
\max_{T, w, e} Eu^P = p(e) (V - w) - T,
\]

(5)

such that the incentive constraint holds:

\[
e \in \arg \max_{e \in [0, \bar{e}]} \{ p(e) w + B + T - K(e) - c(e) \},
\]

(6)

the participation constraint holds:

\[
p(e) w + B + T - K(e) - c(e) \geq B + \phi^A,
\]

(7)

and the budget constraint holds:

\[
B + T \geq K(e).
\]

(8)

Expression (5) reflects the fact that the principal’s expected net return is the difference between the expected surplus from the project and the rent that accrues to the agent, in the form of the bonus \( w \) and the up-front payment, \( T \). Condition (6) is the incentive constraint, which ensures that the effort level provides the maximal payoff to the agent. Condition (7) ensures the participation of the agent, and (8) guarantees that the agent is not required to bear a monetary cost of effort that exceeds his budget and the up-front payment.

For the ease of the exposition, we assume that the two cost functions are such that \( K(e) \equiv (1 - \psi)e \) and \( c(e) \equiv \psi e \), with \( \psi \geq 0 \). When \( \psi = 0 \), exerting effort induces a purely monetary cost, \( K(e) \equiv e \) and \( c(e) \equiv 0 \). The effort level of the agent and the up-front payment received from the principal are linked through the budget constraint 8 that writes \( B + T \geq e \). In this polar case, the problem is purely a problem of monetary investment. Unlike most models of financial contracts, which consider adverse selection problems and focus on issues of risk sharing, the focus here is on problems raised by potentially endogenous monetary investments in a moral
hazard setting. When $\psi = 1$, exerting effort induces a purely non-monetary cost (there is no monetary cost), $c(e) \equiv e$ and $K(e) \equiv 0$. In this polar case, the effort level of the agent and the up-front payment received from the principal are not linked through the budget constraint 8 that writes $B + T \geq 0$. However, the principal cannot ask an up-front payment from the agent larger than the agent’s wealth, $-T \leq B$. In this second polar case, the problem is a moral hazard problem where the effort induces non-monetary costs and the agent’s limited wealth restricts the set of up-front payment levels that the principal may ask. This situation is similar to the problem analyzed in Lewis and Sappington (2000a,b, 2001).

**Example 1:** The following example will be used in order to provide a simple illustration of some of the general results. The probability of success is $p(e) \equiv \frac{1}{\gamma} e^\gamma$ with $\gamma \in (0, 1)$, $e$ belongs to $[0, 1]$ and $V$ belongs to $(0, 1]$. The non monetary cost of effort is zero, $\psi = 0$.

In the next section we develop the analysis of the model introduced and provide the main results.

4 Analysis of the Model

We now proceed with the analysis of the model. Let us first provide the benchmark of the first best situation.

4.1 First benchmark: the first-best situation

The first best effort is the level of effort that maximizes the joint payoff of the agent and the principal. If the project is implemented, the joint payoff is the expected return of the project, $p(e) V$, net of the cost of effort ($K(e) + c(e) = e$):

$$\max_{e \in [0,V]} W = p(e) V - e$$

The solution is given by

$$e^{FB} = g(V),$$

where $g(x) = (p')^{-1}(1/x)$, with $p' > 0$. The effort level, $e^{FB}$, is thus increasing in the value of the project, $V$.\(^8\)

In order to exclude cases in which the first best situation leads to no contract, we make the following assumption:

**Assumption (A2):** The project is worthwhile if the effort level is $e = e^{FB}$:

$$p(e^{FB}) V - e^{FB} > \phi^A + \phi^P$$

This condition also ensures that the first-best level of effort is $e^{FB}$, as defined by (10).

\(^8\)An analysis of financial contracts within a moral hazard setting is provided in Section 6 in Bester (1987). It differs notably from the present polar case, as it assumes perfect competition between principals, risk-averse agents, and exogenous monetary investment.

\(^9\)Indeed, differentiating condition (10), we have $\partial e^{FB} / \partial V = -1 / (V^2 p''(e^{FB})) > 0$. 

4.2 Second benchmark: Limited wealth and symmetric information

Now assume that the principal may offer a contract to the agent in an environment where there is no asymmetry of information, that is, where the effort supplied by the agent is verifiable or at least observable. In this case a contract offered to the agent may include a transfer, $T^v$, a bonus, $w^v$, and a prescribed effort level, $e^v$. The principal will choose the contract that both maximizes his expected payoff and ensures that the participation and budget constraints hold.

If the principal does not offer a contract, he gets his outside option, $\phi^P$. If he decides to offer a contract to the agent, his problem is as follows:

$$\max_{T,w,e} E u^P = p(e)(V - w) - T,$$

such that the participation constraint holds:

$$p(e) w + B + T - e \geq B + \phi^A,$$

and the budget constraint holds:

$$B + T \geq (1 - \psi)e.$$

One can easily show that the principal finds it optimal to require that the agent provides the first-best effort level, $e^v = e^{FB}$ and that a simple optimal contract consists in $w^v = 0$ and $T^v = e^{FB} + \phi^A$. To summarize, the principal requires that the agent provides the first-best effort level, retains full ownership and pays a transfer to the agent equal to the sum of the monetary cost, the non-monetary cost and the value of the outside option. Notice that the agent’s participation constraint (13) is then binding, while the budget constraint (14) is not. Moreover, the principal gets the full surplus of the relationship.\(^\text{10}\)

4.3 Optimal Contract: Motivating versus Funding

We now consider the most interesting situation: the effort is non contractible, and the agent faces a budget constraint. We first focus on the optimal contract when the agent’s wealth is sufficiently high. In this situation, we obtain the following result:

**Proposition 1:** If the wealth of the agent is sufficiently high, $\bar{B} < B$, the optimal contract has the following properties:

(i) Full returns, i.e. the bonus for success equals the value of the project:

$$\hat{w} = V,$$

(ii) No funding, i.e. the up-front payment is always a payment from the agent to the principal:

$$\hat{T} = \phi^A - (p(e^{FB})V - e^{FB}) < 0 < T^v,$$

\(^\text{10}\)In fact, any pair $(T,w)$ such that $T = e^{FB} + \phi^A - p(e^{FB})w^v$ and $w < \frac{B + e^{FB} + \phi^A}{p(e^{FB})}$ is optimal.
(iii) No rent, i.e. the agent gets no surplus:

\[ \widehat{E}w^A - (\phi^A + B) = 0, \]

(iv) Efficiency, i.e. the agent’s effort equals the first best level:

\[ \hat{e} = e^{FB}, \]

and \( B = p(e^{FB})V - \psi e^{FB} - \phi^A. \)

Proposition 1 states that if the agent has sufficient wealth, the principal can sustain efficiency and get the full expected surplus of the project. He can reach this outcome by making the agent the residual claimant for all returns from the project after receiving an up-front payment from the agent equal to the expected surplus of the project.

In the remainder of the analysis we will use the elasticity of the probability function \( p \) and the elasticity of its derivative \( p' \), which are functions of the effort level \( e \):

\[ \epsilon_p(e) = ep'(e)/p(e) \text{ and } \epsilon_p'(e) = ep''(e)/p'(e) \]  \hspace{1cm} (15)

In order to ensure the uniqueness of the optimal contract in the cases we analyze next, we make the following assumption:

**Assumption (A3):** The ratio of the elasticity of the marginal probability of success to the elasticity of the probability of success, \( \epsilon_p'(e)/\epsilon_p(e) \), is non increasing.

This assumption is identical to Assumption 1 in Lewis and Sappington (2000a), and implies that the production technology exhibits diminishing returns to profit sharing.

We can then show the following result:

**Lemma 1:** If the wealth of the agent is intermediate, \( B \leq B < \bar{B} \), the optimal contract is such that: the bonus for success, \( w^* \), is unique and characterized by

\[ 2 - \psi = Vp'(g(w^*)) + \frac{\epsilon_p'(g(w^*))}{\epsilon_p(g(w^*))}, \]  \hspace{1cm} (16)

the up-front payment from the principal to the agent is

\[ T^*(B) = (1 - \psi)g(w^*) - B, \]

and the agent’s effort level is

\[ e^* = g(w^*), \]

and \( \bar{B} = \phi^P - p(g(w^*)) (V - w^*) + (1 - \psi)g(w^*) < B = p(g(w^*))w^* - \psi g(w^*) - \phi^A < \bar{B} \), where

\text{11} We also need the following Inada conditions. \( \lim_{e \to 0} \frac{V}{z - \psi} p'(e) + \frac{1}{z - \psi} \frac{\epsilon_p'(e)}{\epsilon_p(e)} > 1 \) and \( \lim_{e \to \infty} \frac{V}{z - \psi} p'(e) + \frac{1}{z - \psi} \frac{\epsilon_p'(e)}{\epsilon_p(e)} < 1. \) These conditions hold for instance in the case of Example 1, since \( \lim_{e \to 0} \frac{V}{z - \psi} p'(e) + \frac{1}{z - \psi} \frac{\epsilon_p'(e)}{\epsilon_p(e)} = +\infty > 1 \) and \( \lim_{e \to \infty} \frac{V}{z - \psi} p'(e) + \frac{1}{z - \psi} \frac{\epsilon_p'(e)}{\epsilon_p(e)} = \frac{V}{z - \psi} + \frac{\gamma - 1}{z - \psi} < 1 \) because \( V \) and \( \gamma \) are smaller than 1.
Lemma 1 characterizes the optimal contract for intermediate values of the agent’s wealth. Without more structure on the probability $p$, we cannot explicitly characterize the optimal bonus, $w^*$. In Example 1, one can compute the explicit expression of the optimal bonus, $w^* = \frac{V}{1+\gamma}$.\(^{12}\)

For the ease of exposition, we now introduce the following assumption:

**Assumption (A4):** The budget thresholds $\underline{B}$ and $\overline{B}$ are such that $0 < \underline{B} < \overline{B}$.

Assumption (A4) ensures the existence of all feasible cases, it holds in the rest of the paper. We can now derive the properties of the optimal contract:

**Proposition 2:** If the agent’s wealth is intermediate, $\underline{B} \leq B < \overline{B}$, the optimal contract has the following properties:

(i) Shared returns, i.e. the bonus for success, is strictly positive and smaller than the value of the project:

$$0 < w^* < \frac{V}{2 - \psi}.$$  

(ii) Funding, i.e. the up-front payment can be a payment from the principal to the agent when the agent’s wealth is small enough (this payment is, however, bounded from above, $T^*(B) < T^*$). Formally, if $T^*(\underline{B}) \leq 0$ then $T^*(B) \leq 0$ for all $B \in [\underline{B}, \overline{B})$ and if $T^*(\underline{B}) > 0$ there exists $\tilde{B} \in (\underline{B}, \overline{B}]$ such that

$$T^*(B) > 0 \iff B < \tilde{B}.$$  

(iii) Positive rents, i.e. the agent gets a strictly positive surplus:

$$Eu^*_A - (\phi^A + B) > 0.$$  

(iv) Inefficiency, i.e. the effort of the agent is strictly lower than the first best effort level:

$$e^* < e^{FB}.$$  

Proposition 2 states that if the agent’s wealth is intermediate, the principal cannot sustain efficiency and he cannot get the full expected surplus of the project. If the principal chooses to let the agent be the residual claimant of the full returns of the project, $w = V$, the latter makes the first best effort level, then the principal cannot ask for an up-front payment that is larger than $T = (1 - \psi)e^{FB} - B < 0$, and the agent gets a large rent $Eu^{A*} = p(e^{FB})V - \psi e^{FB}$. Instead, the principal can reach a second best by letting a share of the returns of the project to the agent after he has already provided the agent with an up-front payment in order to enable him to exert this second best effort level. Notice that point (iv) ensures that $\overline{B} < \overline{B}$.

In the polar case where the cost of effort does not induce monetary costs ($\psi = 1$), the up-front payment is always a transfer from the agent to the principal ($T^* = -B < 0$). Indeed, in

\(^{12}\)For this example, the optimal effort of the agent is $e^* = \left(\frac{1}{1+\gamma}V\right)^{\frac{1}{\gamma}}$.  

\[g(x) = (p')^{-1}(1/x).\]
this case, the agent does not need money in order to exert the required effort level, the principal then asks the agent to pay up-front an amount that equals the agent’s wealth, $\tilde{T} = -B$. The possibility that the up-front payment is a transfer from the principal to the agent ($T^* > 0$) arises when exerting an effort induces some monetary cost ($\psi < 1$).

Figure 1 illustrates the up-front payment from the principal to the agent as a function of the agent’s wealth, in the case where $T^*(B) > 0$. While it is always a payment from the agent to the principal ($T^* < 0$) when the agent’s wealth is high (when $\tilde{B} \leq B$, see Proposition 1), it is a payment from the principal to the agent ($T^* > 0$) when a sharing contract is signed and the agent’s wealth is sufficiently low (when $B$ lies between $\tilde{B}$ and $\bar{B}$, see Proposition 2).

Figure 1: Optimal bonus and transfer to the agent

We now move on to the case where the agent’s wealth is low. We will show that the situation is quite simple here, specifically:

**Proposition 3:** If the agent’s wealth is sufficiently low, $B < \bar{B}$, the principal does not propose a contract, both the principal and the agent get their outside option, $\phi^P$ and $\phi^A + B$, respectively.

Proposition 3 states that, if the agent’s wealth is sufficiently low, it is too costly for the principal to provide funding to the agent, even if the project is worthwhile a priori (see Assumption A2). For instance, if the principal chooses to let the agent be the residual claimant of the full returns of the project, $w = V$, the latter supplies the first best effort level. However, the principal cannot ask for an up-front payment larger than his outside option $T = B - (1 - \psi)e^{FB} < B - (1 - \psi)e^{FB} = \tilde{\phi}^P$, and then he prefers not to offer a contract to the agent. Combined with Proposition 2, an interesting implication of this result is as follows. If the agent can save money before the contract, and if $B < \tilde{B}$, then the agent would have an incentive to save enough so that the endowment $B$ is sufficiently large for the project to be worthwhile for the principal (i.e., $B$). Indeed, Proposition 2 highlights that the agent would then earn positive rents.

This result is quite similar to results of credit rationing in adverse selection models, keeping in mind that there is no risk aversion in the present setting.

---

13See for instance Proposition 6 in Bester (1987).
We now conclude the characterization of the optimal contract with the following result:

**Proposition 4:** If the agent’s wealth is intermediate, \( \overline{B} \leq B < \overline{B} \), the principal offers a contract such that both the budget and the participation constraints are binding. Moreover, the optimal effort level is \( \tilde{e} = g(\tilde{w}) \), the optimal transfer is \( \tilde{T} = (1 - \psi)g(\tilde{w}) - B < T^w \), and the optimal bonus is characterized by equality \( p(g(\tilde{w}))\tilde{w} - \psi g(\tilde{w}) = B + \phi^A \).

Proposition 4 states that for agent’s wealth levels above those for which the optimal contract is characterized by Proposition 2, and below those for which the optimal contract is characterized by Proposition 1, the optimal values of the bonus and of the transfer are fully characterized by the (binding) participation and budget constraints. Moreover, one can show for this case that the optimal bonus, the optimal transfer and the optimal effort level lie in between the optimal values corresponding to the case of Proposition 2 and the case of Proposition 1. As illustrated in Figure 1, we have \( w^* \leq \tilde{w} \leq \hat{w} = V \) and \( \tilde{T} \leq \hat{T} \leq T^* \). As illustrated in Figure 2, we have \( e^* \leq \tilde{e} \leq \hat{e} = e^{FB} \). Again, in the polar case where the cost is purely non-monetary (\( \psi = 1 \)), the up-front payment is necessarily a payment from the agent to the principal. Indeed, the principal then asks the agent to pay up-front an amount that equals the agent’s wealth, \( \tilde{T} = -B \).

![Figure 2: Optimal effort level of the agent](image_url)

Note: This Figure displays the agent’s optimal effort level as a function of \( B \), in the case of Example 1. We set \( \phi^A = 0 \), \( \phi^P = 7/18 \), \( V = 1 \) and \( \gamma = 1/2 \). The dashed vertical lines correspond to the threshold values \( \overline{B} = 1/18 \), \( \overline{F} = 2/9 \) and \( \overline{P} = 2 \).

The surpluses of the principal and of the agent are characterized in the following Corollary.

**Corollary 1:** The optimal values of the bonus \( w^* \), of the transfer \( T^* \), and of the effort level as well as the surplus of the agent are continuous with respect to \( B \) over \([B, +\infty)\). The surplus of the principal is continuous with respect to \( B \) over \([0, +\infty)\). The surplus of the agent writes:

\[
E w^* - (\phi^A + B) = \begin{cases} 0 & \text{if } B < \overline{B} \\ \frac{B}{\overline{B}} - B & \text{if } \overline{B} \leq B < \overline{B} \\ 0 & \text{if } B \leq \overline{B} \end{cases}.
\]

The surplus of the principal writes

\[
E u^* - \phi^P = \begin{cases} 0 & \text{if } B \leq \overline{B} \\ B - \overline{B} & \text{if } \overline{B} \leq B \leq \overline{B} \\ p(\tilde{e}) V - \tilde{e} - (\phi^A + \phi^P) & \text{if } \overline{B} \leq B \leq \overline{B} \\ p(e^{FB}) V - e^{FB} - (\phi^A + \phi^P) & \text{if } \overline{B} \leq B \end{cases}
\]

and the total surplus of the relationship is given by

13
\[ Eu^{A*} + Eu^{P*} - (\phi^A + \phi^P) = \begin{cases} 
0 & \text{if } B \leq \underline{B} \\
\frac{B - B}{B_0} & \text{if } \underline{B} \leq B \leq \overline{B} \\
p(\overline{e}) V - \overline{e} - (\phi^A + \phi^P) & \text{if } \overline{B} \leq B \leq \overline{B} \\
p(e^{FB}) V - e^{FB} - (\phi^A + \phi^P) & \text{if } \overline{B} \leq B 
\end{cases} \]

Figure 3 illustrates the expected surpluses as functions of the agent’s wealth \( B \). The expected surplus of the agent is zero for \( B \leq \underline{B} \), since he gets his outside option. At \( B = \underline{B} \), the expected surplus of the agent jumps upward and decreases up to \( B = \overline{B} \), because his outside option increases with \( B \). This is so because the optimal contract is such that the agent’s budget constraint is saturated. Over this interval, an additional unit of wealth does not benefit the agent. Above \( \overline{B} \), he gets his outside option, first because the optimal contract is such that both the budget and the participation constraints are binding (when \( B \) lies between \( \underline{B} \) and \( \overline{B} \)), and second because the optimal contract reaches the first best and leaves him no rent (when \( B \) lies above \( \overline{B} \)). The principal’s expected payoff equals his outside option when the agent’s wealth is low (\( B \leq \underline{B} \)), then his surplus is zero. His surplus then increases because the (second-best) contract is implemented (when \( B \) lies between \( B = \underline{B} \) and \( \overline{B} \)) and the up-front payment is bounded by the wealth of the agent. At \( B = \overline{B} \), it increases at a lower rate because the participation constraint is binding (and this is true up to \( \overline{B} \)). For larger values of the agent’s wealth, the budget constraint is no more binding, the optimal contract enables to reach the first-best situation, and the principal gets the full surplus of the relationship, which does not depend on the agent’s wealth.

**Figure 3: Expected surpluses of the principal and of the agent**

Note: This Figure displays the expected surpluses of the principal and of the agent as functions of \( B \), in the case of Example 1. We set \( \phi^A = 0 \), \( \phi^P = \frac{7}{18} \), \( V = 1 \) and \( \gamma = \frac{1}{2} \). The dashed vertical lines correspond to the threshold values, \( \underline{B} = \frac{1}{18} \), \( \overline{B} = \frac{2}{9} \) and \( \overline{B} = 2 \).

### 4.4 Complementarity between Incentives and Transfer

In this section, we ask whether providing incentives may go hand in hand with providing means to the agent. Given that the optimal contract stipulates, in some cases, an up-front payment to the agent and a bonus payment, we go further and ask whether the transfer and the bonus vary in the same or in opposite directions as the value of the project increases.

**Proposition 5:** (i) If the agent’s wealth is sufficiently high, \( \overline{B} \leq B \), that is, when the contract is
efficient, the optimal contract is such that incentives and transfer to the agent are “substitutes” as regards the value of the project: incentives increase, and transfer decreases, as the value of the project increases. Formally, $\frac{\partial \hat{w}}{\partial V} > 0$ and $\frac{\partial \hat{T}}{\partial V} < 0$.

(ii) If the agent’s wealth is intermediate, $\underline{B} \leq B < \overline{B}$, the optimal contract is such that incentives and transfer to the agent are “complementary” as regards the value of the project: incentives increase if and only if the transfer increases as the value of the project increases. Formally, as long as $\psi < 1$, $\frac{\partial \hat{w}^*}{\partial V} > 0$ if and only if $\frac{\partial \hat{T}^*}{\partial V} > 0$ (if $\psi = 1$, then $\frac{\partial \hat{T}^*}{\partial V} = 0$).

(iii) Otherwise, if $B < \underline{B}$ or $\overline{B} \leq B \leq \overline{B}$, the optimal contract is such that incentives and transfer to the agent are independent from the value of the project. Formally, $\frac{\partial \hat{w}^*}{\partial V} = 0 = \frac{\partial \hat{T}^*}{\partial V}$.

This result reveals a striking difference between the case in which the principal can reach an efficient contract, as in the standard moral hazard model when the agent and the principal are risk neutral, and the case in which the principal implements a sharing contract and the participation is not binding (case ii). In the former case, the bonus increases, while the transfer from the principal to the agent decreases, with the value of the project. In the latter case, the bonus and the transfer either both increase or both decrease with the value of the project as long as the effort induces some monetary cost ($\psi < 1$). The intuition of this result is as follows. The solution of the incentive constraint is such that the effort is increases with the bonus, $e = g(w)$ with $g' > 0$. Moreover, when the budget constraint binds, the effort is also an increasing function of the up-front transfer, $e = (B + T)/(1 - \psi)$. As a result, the bonus and the up-front transfer must be positively related $(T = (1 - \psi)g(w) - B)$. In the case where there is no monetary cost ($\psi = 1$), the up-front payment does not depend on the value of the project as it equals (minus) the agent’s wealth. This implies that, when funding is optimal (see Proposition 2, point ii), then an increase in the value of the project has the same qualitative effect on both funding and incentives.

5 Endogenous Agent’s Budget

In this section we investigate whether and to what extent the agent, who anticipates the contractual arrangement, can take advantage of his limited wealth. In order to do so, we analyze whether and to what extent the agent can increase his rents by consuming his wealth before the relationship takes place.

To this purpose, we assume that there are two periods: An initial period $t = 0$ followed by period $t = 1$, this last period corresponding to the game described and studied in Section 3. At period 0, the agent, who has an initial (limited) budget $B_0 \geq 0$, may choose to consume part of his initial budget $b_0 \in [0, B_0]$ and gets a payoff $u^A_0 = b_0$. At the beginning of period 1, the budget of the agent is the remaining share $B = B_0 - b_0$ and the game played is the same as in Section 3.

At period 0, the agent may choose to consume his initial wealth, $B_0$, or to save it for period 1. The agent maximizes the sum of his expected gains:

$$\max_{b_0 \in [0, B_0]} U^A = u^A_0 + \delta E u^{A*},$$
where \( u^A_0 = b_0 \) and \( B = B_0 - b_0 \). Parameter \( \delta \) denotes the agent’s discount factor, and it is assumed to strictly lie between 0 and 1.

This problem can be written by substituting \( b_0 = B_0 - B \):

\[
\max_{B \in [0, B_0]} U^A = B_0 - B + \delta E u^A^* ,
\]

This problem is well defined if \( E u^A^* \) is uniquely defined, i.e. (16) characterizes a unique bonus \( w^* \). In the following, we assume that the optimal bonus \( w^* \) is unique. Notice that, as already mentioned, \( w^* \) is, for instance, unique when \( p(e) = \frac{1}{\gamma} e^\gamma \) with \( \gamma \in (0, 1) \). We also assume that \( B > 0 \). We then obtain the following result:

**Proposition 6:** (i) If the agent is sufficiently wealthy initially, i.e. \( B_0 \geq B \), then he chooses \( B^* = B \) when \( \delta \geq B/B \) and \( B^* = 0 \) otherwise. (ii) If the agent is not sufficiently wealthy initially, i.e. \( B_0 < B \), then he chooses \( B^* = 0 \).

When the agent is sufficiently wealthy initially (case (i)) and sufficiently patient (\( \delta \geq B/B \)), he consumes some of his initial wealth as long as his remaining budget is sufficient for the principal to offer him a sharing contract in period 1. When the agent is not sufficiently patient (\( \delta < B/B \)), since he heavily discounts the future, he prefers to consume all his initial wealth during period 0, which implies that no contract is signed in period 1. When the agent is not sufficiently wealthy initially (\( B_0 < B \)), he anticipates that no contract will be signed in period 1 and, since he discounts the future, he consumes all his initial wealth during period 0.

Regarding the features of the optimal contract in this setting, we obtain:

**Proposition 7:** When the agent’s budget is endogenous, as long as the agent is sufficiently wealthy initially (\( B_0 \geq B \)) and sufficiently patient (\( \delta \geq B/B \)), the equilibrium has the following properties:

(i) Sharing Contract, i.e. if a contract is signed, it is necessarily the sharing contract described in Lemma 1, and it possesses the properties described in Proposition 2.

(ii) Agent’s Full Bargaining Power, i.e. the agent always gets the full surplus of the relationship: \( E u^A^* - (B + \phi^A) = B - B \) and \( E u^P^* - \phi^P = 0 \).

Proposition 6 provides two striking results that diverge from the standard properties of the optimal contract under moral hazard with a risk neutral agent (which is described in Proposition 1). Considering endogenously limited wealth modifies the main properties of the optimal contract drastically. The agent is no more the full residual claimant of the returns of the project, he gets a share of the returns of the project. The up front payment can be a payment from the principal to the agent (if \( T^* (B) > 0 \)), instead of being a payment from the agent to the principal (\( T^* (B) < 0 \)). Moreover, the agent gets all the surplus of the relationship instead of getting no rent. The intuition of this result is as follows. The agent has incentives to keep a sufficiently high budget level (higher than \( B \)) so that the principal will not choose his outside option. He also has incentives to keep his budget sufficiently low (lower than \( B \)) so that the principal will not choose an efficient contract that leaves no rent to him. Since all the contracts that meet these two conditions generate the same total surplus (see Corollary 1), the agent keeps the level
of budget that maximizes the up-front payment he receives from the principal. He then chooses the lowest level, $B_1$, which implies that the principal gets his outside option while the agent gets all the surplus.

6 Examples and Policy Implications

We provide four empirical applications corresponding to problems that have attracted increasing interest among economists in different fields. Specifically, we first present an application on the issue of venture capital, which is the object of a large number of contributions (see Hellman 1998 for a related reference). We then provide an application to franchising, and discuss how some of our results can be tested in this case. A third application is provided on the conservation of natural resources, specifically on the design of payment for environmental services (see Alix-Garcia and Wolf 2014 for a survey on the issue of payments for forest conservation). The last discussion focuses on the means to motivate an agent to raise his maximal potential effort level, which has intuitive meaning and is related to a current debate on the relationship between wealth and cognitive functions (Many et al., 2013; Carvalho et al., 2016). In each case we first describe the problem at hand, highlight how it can be consistent with the model presented in Section 3, and discuss some policy implications that follow from the analysis.

6.1 Venture Capital

In issues of venture capital, funding is provided by investors to startup firms and small businesses often characterized by difficult access to capital markets and long-term growth potential, such as startups producing innovative technologies. The problem here is one of providing money to an entrepreneur or startup business by a venture capital firm at an early stage of development. By doing so, the venture capitalist expects sufficient returns on his investment. The interested reader is referred to Metrick and Yasuda (2010) for an extensive overview of issues related to venture capital.

We here consider the case of a venture capitalist (the principal) who is willing to provide extra financial means to allow an entrepreneur (the agent) to make an investment in a project, in a situation where the agent can also invest his own wealth in the project. This project consists in starting a business which, if successful, generates a certain benefit. The moral hazard problem comes from the fact that the entrepreneur can choose the level of investment, which is constrained by his budget level (the sum of his wealth and of funding provided by the venture capitalist). Instead of investing in the business, the entrepreneur may shirk or invest in perks at the expense of productive investments (see Mehta 2004 for a related discussion). If the entrepreneur accepts the contract offered by the venture capitalist, he receives a bonus payment if the project is successful and nothing otherwise. The probability that the entrepreneur succeeds in developing the business will depend on the level of investment (in material means, training and learning). If the agent rejects the contract offered, he gets his outside option, which yields a payoff equal to the sum of his initial wealth and a given extra profit that he derives from another economic activity. Assuming that the money not invested in the business is spent in added consumption, this description of the venture capitalist’s problem fits with the model developed in Section 3.
In such cases, the implications of our findings allow to provide a new perspective on venture capital contracts. They actually complement Lewis and Sappington (2000b), as our model provides further potential explanations about why venture capitalists typically maintain a significant ownership stake in the ventures they finance (Sahlman, 1990), and why venture capitalists will not always provide large share of proceeds to capable entrepreneurs. In such a situation, when the entrepreneur has sufficiently limited wealth and may need to make extra investment to improve his productivity, then a sharing contract will emerge provided that the entrepreneur’s wealth is sufficiently (but not too) low. We also show that the venture capitalist will then provide both an up-front payment and a revenue sharing contract, but will retain significant stake in the venture.

Our model also provides empirically testable results in the context of venture capital. A first testable prediction is that an increase in the (potential) value of the project has the same qualitative effect on both the level of funding offered by the venture capitalist and the stakes of the entrepreneur. Another testable prediction is that the level of funding decreases, while the entrepreneur’s share increases, with an increase in the entrepreneur’s financial means (as illustrated in Figure 1).

### 6.2 Franchising

In franchise contracts, the franchisee (the agent) benefits from the franchise brand-name, while franchisers are concerned that the actions of franchisees do not deteriorate the reputation of the brand name. For example, the franchisee may reduce his production costs by producing lower quality products at the expense of the brand name’s reputation (Brickley and Dark, 1987). A typical franchise contract includes both a franchise fee, at the onset of the relationship (an up-front payment to the principal), and a royalty rate. The franchisee keeps the remaining share, which is equivalent to receiving a bonus. In our model, this kind of contract emerges as the optimal contractual arrangement when the agent has a sufficient (but not too high) level of wealth.

Moreover, franchising may offer an opportunity to test some of our predictions empirically. A recent paper by Fan et al. (2016) suggests that franchising probability decreases when financial means decrease. More precisely, they show that low collateralizable housing wealth delays chains’ entry into franchising. This is consistent with our results, since we find that when the agent has sufficient wealth, he pays an up-front payment to the agent and the principal retains a share of the output (provided the level of wealth is not too high). A second testable prediction of the model is that the franchise fee increases and the royalty rate decreases with an increase in the franchisee’s financial means (as illustrated in Figure 1). Finally, one could test the prediction that a variation in the potential returns of the business has the same qualitative effect on both the franchise fee and the royalty rate.

### 6.3 Payment for Environmental Services (PES)

The model can be reinterpreted in the context of PES as follows. A forest owner contracts with the government on a given share of land over a given period of time. The government values the plot of land, and the value of the project corresponds here to the government benefit if the
plot of forest is conserved during the period of time (its value is zero if the plot of forest is not conserved). Ex-ante, there is a given number of trees on this land, which is (imperfectly) measured thanks to satellite imagery or to an expert visual evaluation. The contract stipulates that the forest owner receives an up-front payment and gets an additional payment if the plot of land under contract is detected as conserved at the end of the period.\textsuperscript{14} Some trees can be destroyed because of accidental fire or unexpected tree disease, and some trees can also be cut by the forest owner.\textsuperscript{15} It may be difficult and costly to assess whether trees were destroyed because of natural hazard or because of the owner’s will. The owner chooses the number of trees that he cuts during the given period of time, and the remaining trees are conserved: he thus chooses the magnitude of his conservation effort. The owner gets a benefit from cutting trees, and must get incentives to conserve any number of them. The probability that the plot of land is declared to be “conserved” (thus, that the outcome of the project is successful) increases with the number of trees conserved. A typical owner is often financially constrained: he has a limited level of wealth, but needs to use some amount of money in order to live during the period covered by the contract. The owner may also need to switch to other economic activities (e.g. intensive farming), which requires buying capital (production technologies) and/or acquiring specific knowledge related to these economic activities. If the owner does not accept the contract, he cuts the forest and consumes his wealth.

Within this setting, some implications of the analysis provided are useful to discuss an important problem concerning policy design. Indeed, since forest owners face monetary opportunity costs, their level of wealth is constrained and may restrict the range of conservation actions they can undertake. As explained in Jayachandran (2013), a major drawback of this instrument is that it does not take into account the short-term large opportunity costs that liquidity constrained forest owners may face. However, Alix-Garcia and Wolff (2014) argue that contracts vary from 5 years in Mexico to 20 years in Ecuador and that PES programmes tend to pay at the end of each contract year and not at the end of the contract. In the present setting, there is a possibility that wealth-constrained agents might receive funding at the outset of the relationship. Our analysis provides theoretical arguments that support the statement developed in Jayachandran (2013). It highlights that it is actually optimal to offer a contract stipulating revenue sharing on one hand and, on the other side, a positive transfer at the outset of the relationship from the principal to agents characterized by sufficiently low levels of wealth. In other words, the forest owners’ income levels should be used in policy design.

6.4 Wealth and Cognitive Capacity

The topic of the present paper is also related to a current debate on the relationship between wealth and cognitive abilities. There is no consensus on this issue at the moment. Many et al. (2013) provide experimental and field evidence on the fact that low levels of wealth may impede cognitive capacities (focusing on the case of sugarcane farmers in India), while Carvhalo et al. (2016) do not find such a relationship in the case of a group of low-income US households. Even though they provide different conclusions, these two studies suggest that more research is

\textsuperscript{14}Regarding end-of-period payments, we refer the reader to Munoz-Pina et al. (2008) for a description of PES implemented in Mexico.

\textsuperscript{15}For instance, in Brazil, farmers convert forests into cattle pasture land (Simonet et al., 2015).
required regarding the effect of the feeling of (material) scarcity on cognitive functions. While we do not take a stance on this debate, we would like to stress that this topic might constitute a potential application of the present model. Here we want to highlight the potential trade-off that may emerge between providing incentives to an agent to supply effort and aiding him to improve the maximal effort level he can potentially supply.

In this application, the agent’s capability corresponds to the different levels of effort he can supply. It is measured by the length of the interval starting from no effort to a maximal effort level. The higher this maximal effort is, the higher is the agent’s capability. Suppose that this maximum effort level depends on health care related expenses, training or other activities, which have a monetary cost. On the other side, the agent may choose to supply any level of effort within the set of potentially feasible ones, and the cost from supplying effort increases with the effort level.

In such a context, a principal may provide money to the agent to enable him to undertake the above mentioned activities, so that he can supply a higher maximal effort. The other assumptions are the same as in the model presented in Section 3. The important point here is that the agent, when choosing the effort level that he will supply, must satisfy the constraint that it cannot exceed the maximal effort level that is available to him.

Even within this simple specification, implications can be drawn from the present analysis at several levels. First, the analysis interpreted in this context highlights that the principal’s interest may be to aid the agent to increase his capability. A first implication is thus that the present findings provide support to the use of investments in health care related activities, on-the-job training... Moreover, they highlight that incentives and aid are sometimes simultaneously increasing in the value of the project, which is important for policy design.

Finally, on a quite different perspective, and coming back to the application provided in Many et al. (2013) on the case of sugarcane farmers in India, one may notice that our findings offer a new explanation for the prevalence of sharecropping contracts in agriculture (for other explanations see, among others, Eswaran and Kotwal 1985 and Allen and Lueck 2004) based on limited cognitive capacities. In our setting, revenue sharing will emerge every time that the agent’s wealth is sufficiently (but not too) low.

7 Conclusion

We introduce in this article a moral hazard problem where the agent has limited wealth, which constrains his feasible range of actions. This is consistent with several situations where, among others: the opportunity cost is monetary, the effort provided by the agent actually consists in an investment, or the agent can invest in training activities in order to improve his capability. In such cases, the lower the level of wealth is (including transfer from or to the principal), the lower the maximum effort level that can be potentially provided. Bounded wealth and its limiting effect on the set of feasible actions characterizes the distortion with respect to the standard model.

In this setting we show that the optimal contract is, in some cases, a sharing contract and the optimal up-front transfer is a payment from the principal to the agent. Moreover, while a variation in the value of the project has opposite qualitative effects on incentives and
monetary transfer in the case where the agent has sufficient wealth, such a variation has the same qualitative effect on these two variables when the agent has limited wealth. Finally it is also shown that, if the agent can consume some of his wealth at the outset of the contractual arrangement, he gets all the surplus of the relationship.

These findings yield a number of policy implications in the context of (among others) natural resource conservation or venture capital, provides several insights related to a current debate on the potential effect of wealth on cognitive functions, and delivers empirically testable results in the context of franchising and venture capital. For instance, when agents’ income level is low, they suggest the use of Payment for Environmental Services stipulating both revenue sharing and a positive transfer from the principal at the outset of the relationship. Our findings also lead to several testable implications. They predict that funding and the entrepreneur’s share vary in the same direction in venture capital contracts, and that a similar prediction holds for franchise fees and royalty rates in franchise contracts. They also predict that the level of funding decreases, while the entrepreneur’s share increases, with an increase in the entrepreneur’s financial means, and that the franchise fee increases while the royalty rate decreases as the franchisee’s financial means increase.

To conclude, the analysis of the trade off between motivating and funding in a moral hazard setting delivers a range of interesting insights and predictions, even within the simple framework considered here. This article thus constitutes a first step in a broad research agenda. Analyzing the problem in a dynamic setting or when relaxing the assumption that wealth is observable at the time of contracting seem to constitute natural next steps.

Appendix A

We first prove the following preliminary result that will be useful in some of our proofs.

**Lemma 2**: If the principal offers a contract to the agent, i.e. \( p(e)(V - w) - T \geq \phi^P \), then the optimal effort level and the optimal bonus are strictly positive: \( e^* > 0 \) and \( w^* > 0 \).

**Proof of Lemma 2**: Since the principal’s payoff decreases with the up-front payment, the principal has an incentive to decrease the level of the up-front payment as long as the participation constraint (7) and the budget constraint (8) are not binding. Thus, the optimal up-front payment is given by

\[
T^* = (1 - \psi)e - \min \{ B, p(e)w - \psi e - \phi A \}. \tag{17}
\]

The principal’s problem can then be rewritten as follows

\[
Max_{w, e} \widehat{Eu}^P = p(e)(V - w) - (1 - \psi)e + \min \{ B, p(e)w - \psi e - \phi^A \}, \tag{18}
\]

such that

\[
e \in \arg \max_{e \in [0, e^*]} \{ p(e)w - e \}. \tag{19}
\]

We cannot have \( w^* \leq 0 \) or \( e^* = 0 \). Indeed, if \( w^* \leq 0 \) condition (19) implies \( e^* = 0 \). If \( e^* = 0 \) the payoff of the principal becomes \( \widehat{Eu}^P = p(0)(V - w^*) + \min \{ B, p(0)w^* - \phi^A \} \). Since \( \phi^A \geq 0 \), we have \( \widehat{Eu}^P = p(0)V - \phi^A \). Using Assumption (A1), we conclude that \( \widehat{Eu}^P < \phi^P \) and the
principal will not offer a contract to the agent.

**Proof of Proposition 1:** Assume that \( p(e)(V - w) - T \geq \phi^P \) and \( B > p(e)w - \psi e - \phi^A \). Since \( e^* > 0 \), the principal’s problem can be written as

\[
\max_{e, w} \tilde{E}u^P = p(e)(V - w) + p(e)w - e - \phi^A = p(e)V - \phi^A - e,
\]

such that \( p'(e)w = 1 \).

We must have \( p'(e)V = 1 = p'(e)w \). Hence, \( \tilde{w} = V \) and \( \hat{e} = g(V) \). The corresponding payoff of the principal is \( \tilde{Eu}^P = p(g(V))V - \phi^A - g(V) \). The principal prefers to offer a contract because, due to Assumption (A2), we have \( \tilde{Eu}^P = -\tilde{T} = p(g(V))V - g(V) - \phi^A \geq \phi^P \). We must also have \( B > p(g(V))V - \psi g(V) - \phi^A = \overline{B} \). Notice that the principal cannot get a larger payoff if \( B > p(e)w - e - \phi^A \) does not hold.

**Proof of Lemma 1:** Assume that \( p(e)(V - w) - T \geq \phi^P \) and \( B < p(e^*)w^* - \psi e^* - \phi^A \). We know from the proof of Lemma 2 that \( e^* > 0 \), the principal’s problem can then be written as

\[
\max_{e, w} \tilde{E}u^P = p(e)(V - w) - (1 - \psi)e + B,
\]

such that \( p'(e)w = 1 \). Since \( w^* > 0 \) from Lemma 2, we can write \( e = g(w) \). Substituting \( e = g(w) \) into the principal’s objective function, we have

\[
\tilde{Eu}^P = p(g(w))(V - w) - (1 - \psi)g(w) + B,
\]

Let us show that \( w^* < V \). Assume it is not the case, that is \( w \geq V \). Since \( g' > 0 \), the objective function \( \tilde{Eu}^P \) is decreasing in \( w \). This is not compatible with Lemma 2. Hence, \( w^* < V \).

The principal’s problem can then be rewritten as follows:

\[
\max_{w \in [0, V]} \tilde{Eu}^P = p(g(w))(V - w) + B - (1 - \psi)g(w) .
\]

Given that \( \tilde{Eu}^P \) is continuous and differentiable over the compact \([0, V]\) and that the maximum is not reached for \( 0 \) or \( V \), the necessary condition writes:\(^{16}\)

\[
1 - \psi = p'(g(w^*)) (V - w^*) - \frac{p(g(w^*))}{g'(w^*)}
\]

Notice that \( p'(g(w)) = \frac{1}{w} \). Differentiating this condition, we find \( g'(w) = -1/(w^2p''(g(w))) \). We can then rewrite condition (23) as follows:

\[
2 - \psi = Vp'(g(w^*)) + \frac{p(g(w^*))p''(g(w^*))}{(p'(g(w^*))^2}.
\]

\(^{16}\)In the example where \( p(e) \equiv \frac{1}{e^\gamma} \) with \( \gamma \in (0, 1) \), \( e \) and \( V \) belong to \([0, 1]\), the objective function is concave with respect to \( w \).
Condition (24) can be rewritten as follows:

\[ 2 - \psi = Vp'(g(w^*)) + \frac{\epsilon_p'(g(w^*))}{\epsilon_p(g(w^*))}. \]  

(25)

Such a \( w^* \) exists. Indeed, \( w^* \) is necessarily in \([\epsilon, V]\), with \( \epsilon \in (0, V) \) (see the proof of Lemma 1) and since \( p \) is twice continuously differentiable, all the terms in condition (25) are continuous. Hence, using Brouwer’s fixed point theorem, we conclude that condition (25) has a solution as long as the Inada conditions hold. Here, these conditions are \( \lim_{\epsilon \to 0} \frac{\epsilon_p'(g(w^*))}{\epsilon_p(g(w^*))} > 1 \) and \( \lim_{\epsilon \to V} \frac{\epsilon_p'(g(w^*))}{\epsilon_p(g(w^*))} < 1 \) (these conditions hold for instance in the case of Example 1).

The principal’s payoff is \( \widetilde{E}u^{P*} = p(g(w^*)) (V - w^*) - (1 - \psi) g(w^*) + B \), which is larger than \( \phi^P \) if and only if \( B \geq B \).

The initial assumption that \( B < p(e^*) w^* - \psi e^* - \phi^A = \overline{B} \) is without loss of generality. First remark that, if \( B > p(e^*) w^* - \psi e^* - \phi^A \), we can use the proof of Proposition 1 to show that we must have \( B > \overline{B} \), which is a contradiction. Second, let us show that we cannot have an optimal contract such that \( B = p(e^*) w^* - \psi e^* - \phi^A \). We have to show that the expected surplus of the principal is larger when he chooses contract \((\widetilde{w}, \widetilde{T})\) rather than contract \((\tilde{w}, \tilde{T})\) defined in Proposition 4 such that \( p(g(\tilde{w})) = B + \phi^A \) and \( \tilde{T} = (1 - \psi) g(\tilde{w}) - B \). Notice that \( \widetilde{E}u^{P*} = B - \overline{B} \), where \( \overline{B} \) does not depend on \( B \). If the principal chooses contract \((\tilde{w}, \tilde{T})\) then his surplus is \( p(\tilde{e}) V - (1 - \psi) \tilde{e} - \phi^A - \phi^P \) with \( \tilde{e} = g(\tilde{w}) \). In the proof of Proposition 4, we show that the surplus of the principal is continuous at \( B = \overline{B} \). It is then sufficient to prove that the derivative of \( p(\tilde{e}) V - (1 - \psi) \tilde{e} \) with respect to \( B \) is strictly larger than the derivative of \( \widetilde{E}u^{P*} = B - \overline{B} \) with respect to \( B \) for all \( B < \overline{B} \). Using \( p(\tilde{e}) \tilde{w} - \psi \tilde{e} = B + \phi^A \) and \( p'(\tilde{e}) \tilde{w} = 1 \), we have \( \frac{d\tilde{w}}{dB} = \frac{1}{p'(\tilde{e}) + g'(\tilde{w})} \). Differentiating \( p(\tilde{e}) V - (1 - \psi) \tilde{e} \) with respect to \( B \), we find

\[ \frac{Vp'(\tilde{e})}{p'(\tilde{e}) + g'(\tilde{w})} > 2 - \psi. \]

(26)

This inequality holds for \( B < \overline{B} \). Indeed, when \( B = B \), \( \tilde{e} = e^* \) and then the left hand side equals 1. By assumption (A3) and the fact that \( p'' \leq 0 \), we know that the left hand side in (26) is strictly decreasing in \( \tilde{e} \), which in turn is strictly increasing in \( B \).

Hence, if \( \overline{B} \leq B < \overline{B} \), then the principal proposes \( w^* \) characterized by (24) and \( T^* = g(w^*) - B \).

**Proof of Proposition 2:** We know from Lemma 2 that \( w^* > 0 \). Using condition (16) and the fact that \( g'(w^*) > 0 \), we conclude that \( w^* < V/(2 - \psi) \), which proves point (i). Now let us prove point (ii). From the proof of Lemma 1, we know that \( T^* = (1 - \psi) g(w^*) - B \), which is strictly positive if and only if \( (1 - \psi) g(w^*) > B \). Since we must have \( \overline{B} \leq B \), we conclude that \( T^* \leq 0 \) when \( (1 - \psi) g(w^*) \leq B \), i.e. \( 0 \leq \phi^P - p(e^*) (V - w^*) \). When \( \phi^P - p(e^*) (V - w^*) < 0 \), we have \( B < (1 - \psi) g(w^*) \), which is sufficient to prove point (ii). Now let us consider point (iii). The agent’s payoff is \( E u^{A*} = p(e^*) w^* + B + T^* - (1 - \psi) e^* = p(e^*) w^* \). From the proof of Lemma 1, we know that the participation constraint (7) is not binding, thus we have \( E u^{A*} > B + \phi^A \). Let us prove point (iv). We have \( e^* = g(w^*) \) with \( w^* < V \). Since \( g'(w^*) > 0 \), we conclude that
\( e^* = g(w^*) < g(V) = e^{FB} \).

**Proof of Proposition 3:** We have \( B < \overline{B} \). Assume that \( p(e)(V - w) - T \geq \phi^P \). If \( B < p(e^*)w^* - \psi e^* - \phi^A \), we know from the proof of Lemma 1 that the principal’s payoff is \( E u^{P^*} = p(g(w^*)) (V - w^*) - (1 - \psi) g(w^*) + B \), and it is less than \( \phi^P \) since \( B < \overline{B} \). This is a contradiction. If \( B \geq p(e^*)w^* - \psi e^* - \phi^A \), we know from the proof of Proposition 1 that we must have \( B \geq \overline{B} \). Since \( \overline{B} \geq \overline{B} \), we have \( B \geq \overline{B} \), a contradiction.

**Proof of Proposition 4:** We have \( \overline{B} \leq B \leq \overline{B} \). Assume that \( p(e)(V - w) - T \geq \phi^P \). If \( B < p(\bar{e}) \bar{w} - \psi \bar{e} - \phi^A \), we know from the proof of Lemma 1 that we must have \( B < \overline{B} \), which is a contradiction. If \( B > p(\bar{e}) \bar{w} - \psi \bar{e} - \phi^A \), we know from the proof of Proposition 1 that we must have \( B > \overline{B} \), which is a contradiction. Now assume that \( p(\bar{e}) \bar{w} - \psi \bar{e} = \phi^A + B \). Using the incentive constraint, we have \( \bar{e} = \hat{g}(\bar{w}) \) where \( g(x) = (p')^{-1}(1/x) \). Using the budget constraint, we have \( \overline{T} = (1 - \psi)g(\bar{w}) - B \). Moreover, in this case \( \bar{w} \) is characterized by \( p(g(\bar{w})) \bar{w} - \psi g(\bar{w}) = \phi^A + B \). Such a \( \bar{w} \) exists. Indeed, \( \phi^A + B \geq 0 \) and when \( \bar{w} \) gets arbitrarily close to 0, \( p(g(\bar{w})) \bar{w} - \psi g(\bar{w}) \) gets close to 0 and when \( \bar{w} \to +\infty \), then \( p(g(\bar{w})) \bar{w} - \psi g(\bar{w}) \) gets arbitrarily large. It remains to show that the principal’s surplus is non-negative, i.e. \( p(\bar{e})(V - \bar{w}) - \overline{\tilde{T}} \geq \phi^P \).

Let us first show that the principal’s payoff is continuous at \( B = \overline{B} \). When \( B = \overline{B} \), we have \( p(g(\bar{w})) \bar{w} - \psi g(\bar{w}) = \overline{\overline{B}} + \phi^A \), which is equivalent to \( p(g(\bar{w})) \bar{w} - \psi g(\bar{w}) = p(g(w^*))w^* - \psi g(w^*) \). Since \( p \) and \( g \) are increasing functions, we must have \( w^* = \bar{w} \) and then \( T^* = \overline{\tilde{T}} \), \( e^* = \bar{e} \) and then the principal’s payoff is continuous at \( B = \overline{B} \). Now let us show that the principal’s payoff is continuous at \( B = \overline{B} \). In this case, we have \( p(g(\bar{w})) \bar{w} - \psi g(\bar{w}) = \overline{\overline{B}} + \phi^A \), which is equivalent to \( p(g(\bar{w})) \bar{w} - \psi g(\bar{w}) = p(g(V))V - \psi g(V) \). Notice that the derivative of \( p(g(x))x - \psi g(x) \), given that the incentive constraint \( p'(g(x))g(x) = 1 \) always holds, is positive. Hence \( \bar{w} = V, \bar{e} = e^{FB}, \overline{\overline{T}} = T^{FB} \) and then the principal’s payoff is continuous at \( B = \overline{B} \). Now, let us show that \( \bar{w} \) is strictly increasing in \( \bar{B} \). Is is sufficient to notice that \( p(g(\bar{w})) \bar{w} - \psi g(\bar{w}) = \phi^A + B \) where the left hand side is strictly increasing in \( \bar{w} \). We then have \( w^* \leq \bar{w} \leq V \). Finally, let us show that the principal’s payoff is strictly increasing in \( \bar{B} \). It can be written as \( p(g(\bar{w}))V - (1 - \psi)g(\bar{w}) - \phi^A \). The derivative of this expression with respect to \( B \) is given by \( (V/\bar{w} - (1 - \psi))g'(\bar{w}) \frac{\partial \bar{w}}{\partial B} \). We then have \( \frac{\partial \bar{w}}{\partial B} > 0 \) for \( \bar{B} < \overline{B} \) and \( \frac{\partial \bar{w}}{\partial B} = 0 \) for \( B = \overline{B} \). Since the principal’s surplus is strictly positive for \( B = \overline{B} \), it is strictly positive for \( \overline{B} \leq B < \overline{B} \).

**Proof of Corollary 1:** In the proof of Proposition 4, we show that the optimal effort level, the optimal bonus, the optimal transfer and the principal’s surplus are all continuous at \( B = \overline{B} \) and at \( B = \overline{B} \). The surplus of the agent is then continuous at these points too. This is sufficient to conclude the proof (from inspection of the expressions characterizing the effort level, bonus, transfer, and surpluses).

**Proof of Proposition 5:** To prove point (i), it is sufficient to notice that \( p'(e^{FB})V = 1 \) and the result follows. Indeed, since \( \overline{B} \leq B \), then \( \frac{\partial \overline{\overline{B}}}{\partial B} = -p(g(V)) < 0 \) and \( \frac{\partial \bar{w}}{\partial B} = 1 > 0 \). Point (ii) is immediate from Lemma 1. Point (iii) is immediate from Propositions 3 and 4.

**Proof of Proposition 6:** The objective function, \( U^A \), is decreasing in \( B \) on each interval. If \( \overline{B} \leq B \), the agent chooses \( B = \overline{B} \) and then gets \( B_0 = \overline{B} + \delta (\overline{B} + \phi^A) \). If \( B \leq \overline{B} \), the agent chooses \( B = 0 \) and he gets \( B_0 + \delta \phi^A \).
**Proposition 7:** Immediate from the proof of Proposition 6.
# References


